Political Networks, Ideology and Lobbying

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September 15, 2011

Abstract

We study lobbying with information in a setting where politicians are connected through a network describing information flows. There are two opposing lobby groups, who choose which politician to address. The decision depends on the ideological bias of politicians as well as the overall network structure.

We show that there is an interplay between the strength of the network, as given by the information flow, and ideological bias. More precisely, when the information flow in the network is high, the effect of the ideological bias is amplified. Further, the network structure affects whether a priori supporting or opposing politicians are being lobbied. However, it matters not only what ideological biases politicians have, but also whether they are connected with like-minded politicians or politicians with a different ideological bias.

"The real puzzle of its [a lobbies] P[ublic]A[ffairs] management is
to select the most useful actors to approach, the best connections . . . ".

van Schendelen (2010) p. 113.

1 Introduction

Lobbyists think carefully about who they should talk to as can be seen from the given citation. This sentence is taken from one of the many publications that deal with the question of how to lobby. In fact, there exists a vast literature on the right approach to public affairs management, which is another indicator that lobby groups indeed devise strategies to maximise their influence. Another indicator that lobbying is a carefully planned process is the sheer number of lobbyists employed in Washington or Brussels.

Still, there remain some question regarding who is being lobbied, who is the right actor to approach. Lobbyists do not face isolated individuals, but members of parties and committees, politicians that have colleagues, who also have a say on policy issues. This implies that lobby groups can also influence political actors indirectly, through their

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network. In practice, what happens is that lobby groups do not only talk to the main deciders of an issue or to the ones with formal power, but also to politicians who are well-connected and thus adept at spreading the information they obtain among their collegues. In the European Parliament, for example, there exist several so-called inter-groups. Any member of the European Parliament can be a member of an inter-group independent of the party affiliation. To the meetings of inter-groups many lobbyists are invited to provide information about the issues discussed. Then, the members of the inter-group work on putting these issues on the official agenda of the European Parliament. This seems to be a typical case, in which special interest groups give information to politicians, who then disperse it among other politicians. It is also fairly common for firms to lobby delegates of the districts they have plants in. These delegates try to prevent an unfavorable outcome by talking to other delegates. All these arguments indicate that there exists a political network and that politicians need to be seen in the context of this network.

But a politician is probably not fully characterized by his position in the network. Another factor that seems important when thinking about lobbying is the ideological bias of a politician. There is an extensive literature, both theoretical and empirical, on whether like-minded or initially opposing politicians are lobbied. More accurately, there are two competing strands of literature of what the optimal lobbying strategy is in terms of ideology. One strand of literature claims that supporting lobbyists should be addressed, that is those politicians are lobbied that are a priori in favor of a lobby group.[1] The main paper of other strand of literature is probably Austen-Smith and Wright (1992). They develop a theoretical model where opposing or undecided politicians are primarily lobbied. However, if a second lobby group is involved in the lobbying process, then also the supporting politicians are addressed directly. Empirically, it does not seem clear whether lobby groups pursue the one or the other strategy. Austen-Smith and Wright (1994) show that indeed opposing politicians are lobbied, unless there is an opposing lobby group, in which case also the supporting politicians are adressed. This finding is contradicted by Hojnacki and Kimball (1998), who show that the supporting politicians are addressed.

The goal of this paper is to add to this literature and to understand what the optimal lobbying strategy is given that politicians can be ideologically biased and they are connected among each other.

The results I find depend on the network structure among politicians. Also, there is an interplay between the strength of a network and the actual bias of a politician. If the network leads to a high amount of exchange between politicians, then this reinforces the ideological bias of a politician. I can find settings where opposing politicians are lobbied as well as constellations where supporting ones are addressed. Thus, the theoretical model proposed here might be able to reconcile the two aforementioned strands of literature.

The rest of the paper is structured as follows: Section 2 provides a literature review;

[1] For examples of this literature see the introduction of Hojnacki and Kimball (1998).

Section 3 introduces the model, which will be solved in Section 4. Section 5 concludes.

2 Literature Review

2.1 Optimal Targeting

This paper is closely related to that of Lever (2010), who models a voting competition on a network. He is also interested in which politicians are targeted. The approach as well as the focus chosen here, differ from his, though.

In his setting, two lobby groups choose a certain amount of information or money to give to agents within the network. The information or contribution politicians receive affects their preferences for the lobby groups. In a second step, political actors take the opinion of their neighbors in the network dynamically into account. This second step is referred to as the network stage. However, how likely a politician sticks to his original opinion still matters for the outcome. Lever (2010) focuses on majoritarian competition, where he finds that both lobbyists spend the same amount on each politician.

But which politicians are targeted depends on the updating horizon. If political actors do not update through the network, then only the pivotal voters receive contributions. Whenever the time to update approaches infinity, then the contributions are relative to the DeGroot weight of a politician. In this case, there is also a consensus reached, that is everyone has the same opinion at the end of the network stage. Whenever there is an updating process, but the time used for updating does not approach infinity, then both the pivotality as well as the centrality as given by the DeGroot weights matter for the spending strategy.

Lever (2010) also tests whether his findings hold empirically. He finds that both the DeGroot weights as well as the pivotality of a politician matter in explaining who lobbyists contribute to.

One important difference is that in Lever's model introducing ideology does not affect the strategy chosen by the lobbyists. In fact, ideology is irrelevant for the maximization problem whenever the updating horizon of the network stage approaches infinity and when the strategy chosen in equilibrium is a pure strategy. It only implies that there is no consensus reached. In the set up chosen here, ideology is crucial and affects the optimization. It changes outcomes dramatically. As previously mentioned, there exists evidence that ideology indeed matters, such as Austen-Smith and Wright (1994) and Hojnacki and Kimball (1998). In this sense, the approach chosen here can be seen as an extension of Lever (2010) or as having a different focus. Whereas Lever (2010) concentrates on the trade off between pivotality and centrality, I emphasize the relationship between ideology and centrality.

As ideology matters, the results here depend on the network structure. This allows us to make comparisons regarding networks. But it also matters at what position in the network what type of politician is. Comparing different combinations gives useful insights.

Additionally, it can been asked what the role of political fragmentation is. Further, it is possible to look at the effects of polarization in this framework.

Groenert (2010) is also interested in lobbying in networks. She considers a single lobbyist, who tries to influence a network of political actors. A politician initially has an ideological bias in the sense that he is either for or against the lobbyist's proposal. Then, there is an updating stage, where each politician adjusts his opinion depending on the opinion of the direct neighbors: a politician votes in favor of the proposal if the fraction of his neighbors voting in favor of the proposal exceeds an idiosyncratic threshold. She focuses on threshold networks and finds that for this subclass the optimal strategy is to successively lobby the opposing politicians with the highest number of links.

The main difference to my approach is that I have two lobbyists, which allows to look at the strategic interaction. Further, the ideological bias here is more differentiated. In my case there are politicians who are more or less biased or completely neutral.

2.2 Lobbying with Information

I focus here on lobbying with information. This means, I only look at information transmission from lobby groups to political actors. According to Wright (1990), what matters are personal interactions between politicians and lobbyists. Campaign contributions serve to grant access to politicians, but they do not affect political outcomes when controlling for the interaction. Therefore, it seems most important to understand how the process of information transmission works. There are other means which are subsumed under the term lobbying, such as grass root lobbying, involving media etc. These means of influence are not addressed here.

The question, especially with regards to Lever (2010) and Groenert (2010), who both have a model of peer effects, is in how far the lobbying with information differs from a model of peer effects. Peer effects imply that a politician listens to the opinion of the other politician in this setting and is then more likely to choose a certain position as those around him choose the same position. The information transmission approach insists that politician gather information from their peers and then make a decision based on the information they obtain and not based on the others decision. I believe that in many settings, certainly in settings, where important decision are made, peers are used to learn about situations, but that the decision is made because of what is learned from the peers and not simply from mimicking others behavior.[2]

[2]In essence this is an empirical question. Groenert (2010) gives many examples of peer effects such as teenagers are more likely to drop out of high school if their peers drop out. But the question is, do they drop out because they observe that their peers do ok after dropping out of school (an information criterion) or because they drop out (peer effect). Especially, when it comes to the adoption of new technology, I believe that the information is crucial. A farmer will not adopt new seeds because his neighbors do so, but after he sees that the neighbor's crop is larger. But to address this issue in our specific setting, is beyond the scope of this paper.

3 Model

3.1 Setup

A finite set of $N = \{1, \dots, n\}$ politicians are connected in a network described by an undirected graph. A link between two nodes implies that these politicians share information with each other. I assume that the network consists of a single component. [3]

Further, there are two lobbyists, L_1 and L_2, who want to influence this group of politicians to receive a favorable outcome. To be able to influence the network of politicians, a lobbyist has to establish a link with at least one politician. I assume that each lobbyist will always establish a link with exactly one politician. The cost of establishing a link is at this point zero. [4] Both lobbyists decide simultaneously which node they want to connect with.

Once a lobbyist is linked to a politician he can inform this politician about some relevant issue. The information a politician receives from the lobby group is then forwarded to the other politicians within the network along the shortest paths. This assumption does not necessarily imply that information flows only along the shortest paths. It can also interpreted such that information flows along all the paths in the network and information obtained along the shortest path is better than that obtained through other ways. So, in the end, only the information along the shortest path matters. The information is discounted by δ, $0 < \delta < 1$ as it permeates the network. When a lobbyist links with some politician i, the direct neighbours of i receive information discounted be δ, the second-order neighbours obtain information discounted by δ^2 etc. The idea here is that a politician, who learns about a certain issue cannot remember every detail. Therefore, as the information permeates the network, it decays. What should be captured here is that lobbying someone directly is different from lobbying a politician via several intermediaries.

Every politician has some influence on the policy. This is modelled using a success contest function. So, assume that lobbyist 1 chooses some action s^1 and lobbyist 2 s^2, where $s^1, s^2 \in N$. Then lobbyists 1 obtains at node s_i

$$p^1(s^1, s^2|s_i) = \begin{cases} \frac{\varphi_1 \delta^{l(s^1,s_i)}}{\varphi_1 \delta^{l(s^1,s_i)} + (1-\varphi_1)\delta^{l(s^2,s_i)}} & \text{if } s_i \text{ with bias for } l_1 \\ \frac{(1-\varphi_2)\delta^{l(s^1,s_i)}}{(1-\varphi_2)\delta^{l(s^1,s_i)} + \varphi_2 \delta^{l(s^2,s_i)}} & \text{if } s_i \text{ with bias for } l_2 \\ \frac{\delta^{l(s^1,s_i)}}{\delta^{l(s^1,s_i)} + \delta^{l(s^2,s_i)}} & \text{if } s_i \text{ with no bias} \end{cases}$$

[3] The assumption that it is a single component need not hold. Then, the analysis would have to be done for each component.

[4] Incorporating linear costs is straightforward as it does not change the optimization. Therefore, I will introduce costs at a later stage.

whereas the payoff of lobbyist 2 is given by

$$p^2(s^1, s^2|s_i) = \begin{cases} \frac{(1-\varphi_1)\delta^{l(s^2,s_i)}}{\varphi_1\delta^{l(s^1,s_i)}+(1-\varphi_1)\delta^{l(s^2,s_i)}} & \text{if } s_i \text{ with bias for } l_1 \\ \frac{\varphi_2\delta^{l(s^2,s_i)}}{(1-\varphi_2)\delta^{l(s^1,s_i)}+\varphi_2\delta^{l(s^2,s_i)}} & \text{if } s_i \text{ with bias for } l_2 \\ \frac{\delta^{l(s^2,s_i)}}{\delta^{l(s^1,s_i)}+\delta^{l(s^2,s_i)}} & \text{if } s_i \text{ with no bias} \end{cases}$$

where $l(s^1, s_i)$ ($l(s^2, s_i)$) is the number of links between the node lobbyist 1 (2) chooses and the node we consider. The parameters φ_1 and φ_2 give the ideological bias of a politician in favor of L_1 and L_2 respectively. If a politician is biased in favor of a lobby group, then he attaches more weight to the information given by this lobby group as I assume $\varphi_1, \varphi_2 \in (\frac{1}{2}, 1)$. This also implies that a politician's opinion can always be somewhat affected and only then lobbying makes sense. The policy can be interpreted as a lobbyist's share of a budget that is spent among the two lobbyists. In the European Union, for example, there exists a budget for research on energy production. Then one lobbyist might be an institution that works on solar energy, the other one an insitution working on nuclear power. But it might also be interpreted as a policy value where the one lobbyist wants higher value, the other a lower one. An example for this is would be the tax on gasoline. Oil companies or car manufaturers might be for a low tax, environmental groups for a high tax. Then the shares a lobbyist receives can be interpreted as how much the politicians went towards the proposal of the one group. A second possible interpretation is the that $p_1(s^1, s^2|s_i)$ gives the probability of politician i to vote for the preferred outcome of L_1. Note that independent of the ideology all information is forwarded. One can imagine that the politicians truthfully convey all information so as to be credible, but this is nevertheless a stringent assumption. In an extension, I will also make the information transmission dependent on the ideological bias of the politicians.

The payoffs of lobbyist 1 and 2 of the entire network, when both lobbyists play some action, are given by

$$\pi^1(s^1, s^2) = \frac{1}{n}\sum_{i=1}^{n} p^1(s^1, s^2|s_i) \tag{1}$$

$$\pi^2(s^1, s^2) = \frac{1}{n}\sum_{i=1}^{n} p^2(s^1, s^2|s_i) \tag{2}$$

This implies that $\pi^1(s^1, s^2) + \pi^2(s^1, s^2) = 1$. But we are interested in strategies, possibly mixed strategies, which I denote by σ^i, $i \in \{1, 2\}$. The overall payoff for a lobbyist is then a combination of different payoffs that are calculated from (1) for L_1 and from (2) for L_2.

Formally,

$$\Pi^1(\sigma^1, \sigma^2) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma^1(s_i)\sigma^2(s_j)\pi^1(s_i, s_j)$$
$$\Pi^2(\sigma^1, \sigma^2) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma^1(s_i)\sigma^2(s_j)\pi^2(s_i, s_j)$$

Note that $\Pi^1(\sigma^1, \sigma^2) + \Pi^2(\sigma^1, \sigma^2) = 1$.

What is specified here is a game G. It consists of two players, namely lobbyists L_1 and L_2. Both players have the same set of actions $S = \{s_1, s_2, \ldots, s_n\}$, which is finite. The payoffs are $\Pi^1(\sigma^1, \sigma^2)$ and $\Pi^2(\sigma^1, \sigma^2)$.

3.2 Properties of the Payoff Function

Before starting to solve the specified game, I would like to discuss some properties of the payoff function.

Only Relative Distance Matters

$$\frac{\varphi_1 \delta^{l(s^1, s_i)}}{\varphi_1 \delta^{l(s^1, s_i)} + (1 - \varphi_1)\delta^{l(s^2, s_i)}} = \frac{\varphi_1}{\varphi_1 + (1 - \varphi_1)\delta^{l(s^2, s_i) - l(s^1, s_i)}}$$

Independently of whether we have an ideological bias or not, only the relative distance matters. Note that there is an upper bound on the relative distance. As there always exists a path from s^1 to any s_i via s^2 the relative distance in absolute terms can never be larger than the number of links of the shortest path between politicians s^1 and s^2. More formally, let this shortest path be denoted by m. Then,

$$|l(s_i, s_o) - l(s_j, s_o)| \leq m \quad \forall s_i, s_j, s_o \in S.$$

This property makes it possible to split the politicians according to their relative distances. Let $h_k^{s_i s_j}$ denote the number of politicians for which $l(s_i, s_o) - l(s_j, s_o) = k$, where $k \in \{-m, -m+1, \ldots, m\}$. It has to hold that $\sum_{k=-m}^{m} h_k^{s_i s_j} = n$. Last, this property implies that when lobbyists choose the same politician, the discount factor does not matter.

No Ideological Bias: Payoffs Constant in Distances

If there are only unbiased politicians in the network, then

$$\frac{1}{1 + \delta^j} + \frac{\delta^j}{1 + \delta^j} = \frac{1}{1 + \delta^k} + \frac{\delta^k}{1 + \delta^k} = 1, \qquad j, k \in \mathbb{N}.$$

When deciding whom to lobby, moving one step closer to an unbiased node and further away from another one (by the same number of links) does not make a difference in terms of payoffs.

Ideological Bias: Payoffs Affected by Distances

When there is an ideological bias, the previous statement does not longer hold. Instead,

$$\frac{\varphi_1}{\varphi_1 + \delta^j(1-\varphi_1)} + \frac{\delta^j\varphi_1}{\delta^j\varphi_1 + (1-\varphi_1)} \neq \frac{\varphi_1}{\varphi_1 + \delta^k(1-\varphi_1)} + \frac{\delta^k\varphi_1}{\delta^k\varphi_1 + (1-\varphi_1)},$$

where $j \neq k, \quad j, k \in \mathbb{N}$.

4 General Analysis

The aim of this analysis is to find the Nash equilibria of the described game and to characterize them. But to be able to do so it is first necessary to ensure the existence of a Nash Equilibrium.

Proposition 1. *Let $G = \langle \{L_1, L_2\}, S, \{\Pi^1(\sigma^1, \sigma^2), \Pi^1(\sigma^1, \sigma^2)\}\rangle$ be a finite normal form game. Then G has at least one, possibly mixed, Nash equilibrium.*

Proof. See e.g. Fudenberg and Tirole (1991). □

Thus, as existence is given, I can begin to characterize the possible Nash equilibria.

I will focus for now on two network structures, namely the ring and the star. I do this as these two networks can be seen as the two extreme cases and so they highlight the different factors at play. When necessary I will consider some other structures as well.

4.1 Unbiased Politicians

Consider as a baseline the case where there are only unbiased politicians.

Proposition 2. *In a Nash equilibrium, $\Pi^1 = \Pi^2 = \frac{1}{2}$.*

Proof. This follows from the fact that we have a constant sum game. □

Note that this result holds for all network structures in which there are only unbiased politicians.

From this we know that for a Nash equilibrium in pure strategies it has to hold that

$$\begin{aligned} \frac{1}{n}\sum_{i=1}^{n} \frac{\delta^{l(s',s_i)}}{\delta^{l(s',s_i)} + \delta^{l(s^*,s_i)}} &\leq \frac{1}{2} \\ \Leftrightarrow \sum_{i=1}^{n} \frac{1}{1 + \delta^{l(s^*,s_i) - l(s',s_i)}} &\leq \frac{n}{2}, \end{aligned}$$

where s' denotes the deviation from the equilibrium and s^* the equilibrium strategy.

Using the fact that $\sum_{k=-m}^{m} h_k^{s_i s_j} = n$,

$$-h_{-m}^{s^*s'}\frac{1-\delta^m}{2(1+\delta^m)} - h_{-(m-1)}^{s^*s'}\frac{1-\delta^{m-1}}{2(1+\delta^{m-1})} - \cdots - h_{-1}^{s^*s'}\frac{1-\delta}{2(1+\delta)} + h_1^{s^*s'}\frac{1-\delta}{2(1+\delta)} + \cdots + h_m^{s^*s'}\frac{1-\delta^m}{2(1+\delta^m)} \le 0$$

$$\Leftrightarrow \sum_{k=1}^{m} \frac{1-\delta^k}{2(1+\delta^k)}(h_k^{s^*s'} - h_{-k}^{s^*s'}) \le 0.$$

Proposition 3 (Ring). *In a ring with n politicians, any strategy combination defines a Nash equilibrium.*

Proof. Let lobbyist 1 choose s_1 without loss of generality. Due to the symmetry in the circle, $h_k^{s_1 s_j} = h_{-k}^{s_1 s_j}, \forall s_j \in S$. Whenever this holds we have a NE as

$$\sum_{k=1}^{m} \frac{1-\delta^k}{2(1+\delta^k)}(h_k^{s^*s'} - h_{-k}^{s^*s'}) = 0.$$

As L_2 is indifferent between all his actions, he can mix between all of them. The same argument holds for L_1 due to symmetry. Thus, any strategy can be part of a NE. □

What this implies is that when politicians are the same both in terms of ideology and centrality in the network, then the lobbyists are indifferent between lobbying all of them.

Proposition 4 (Star). *In a Star the unique Nash Equilibrium is that both lobbyists connect to the center.*

Proof. Let the strategy of choosing the node in the center be w.l.o.g s_1. It suffices to show that s_1 is a strictly dominant strategy.

Suppose that L_2 connects to a node on the periphery, i.e. he chooses $s_j \neq s_1, s_j \in S$.

Then L_1 can connect to the same node on the periphery, which yields $\Pi^1 = \frac{1}{2}$. If L_1 connects to a different node on the periphery, i.e his strategy is $s_i \neq s_j, s_i \neq s_1, s_i \in S$ this also yields $\Pi^1 = \frac{1}{2}$ as the distance to all nodes in the network is the same except for s_i and s_j. But the advantage for lobbyist 1 at node s_i is the same as the advantage of lobbyist 2 at node s_j and so the overall payoff is $\frac{1}{2}$. Last, if L_1 connects to the center he has an advantage of distance 1 at all nodes except for s_j. His payoff is then $p_1 = \frac{1}{n}((n-1)\frac{1}{1+\delta} + \frac{\delta}{1+\delta}) > \frac{1}{2}$, as $\delta < 1$.

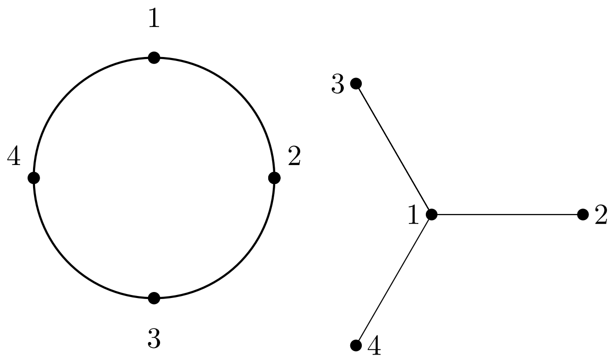
If L_2 instead connects to the center and L_1 connects to a node on the periphery his payoff is $\Pi^1 = \frac{1}{n}(\frac{1}{1+\delta} + (n-1)\frac{\delta}{1+\delta}) < \frac{1}{2}$. So, L_1 is better off connecting to the center.

Due to symmetry of the lobbyists, the same argument holds for L_2. Thus, connecting to the center is a strictly dominant strategy for both lobbyists and therefore the unique Nash equilibrium. □

Thus, in the star, both politician want to lobby the most central politician. This seems intuitive.

But overall, the payoffs from lobbying are the same, independently of the network structure.

Figure 1: Example



(a) Ring (b) Star

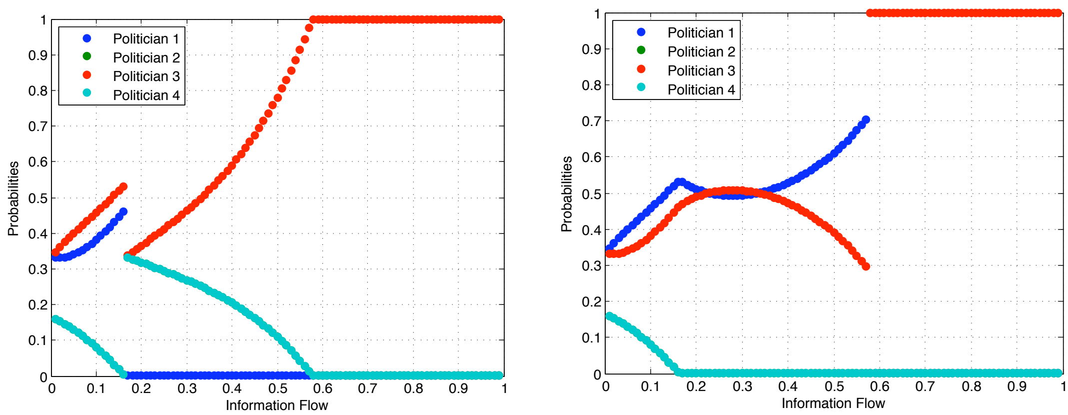
4.2 4 Politicians

Before characterizing the possible networks with n agents, I will give examples when there are only four politicians. This illustrates the importance of centrality, how the distances matter and what the impact of the ideology on the equilibrium strategies is.

Ring: One biased and three neutral politicians

Suppose w.l.o.g. that politician 1 is in favor of L_1. All the other politicians are still neutral. In this case, there are three types of equilibria. [5]

Figure 2: Equilibrium Strategies ($\varphi_1 = \frac{3}{4}$)



(a) Equilibrium Strategy of L_1 (b) Equilibrium Strategy of L_2

In the first type of equilibrium, both lobbyists assign positive probability to all politi-

[5]To clarify, for each parameter constellation there is exactly one type of equilibrium. But overall there are three types.

cians. The probabilities are dispersed as follows:

$$\sigma^1(s_1) = \sigma^2(s_3)$$
$$\sigma^1(s_3) = \sigma^2(s_1)$$
$$\sigma^1(s_2) + \sigma^1(s_4) = \sigma^2(s_2) + \sigma^2(s_4).$$

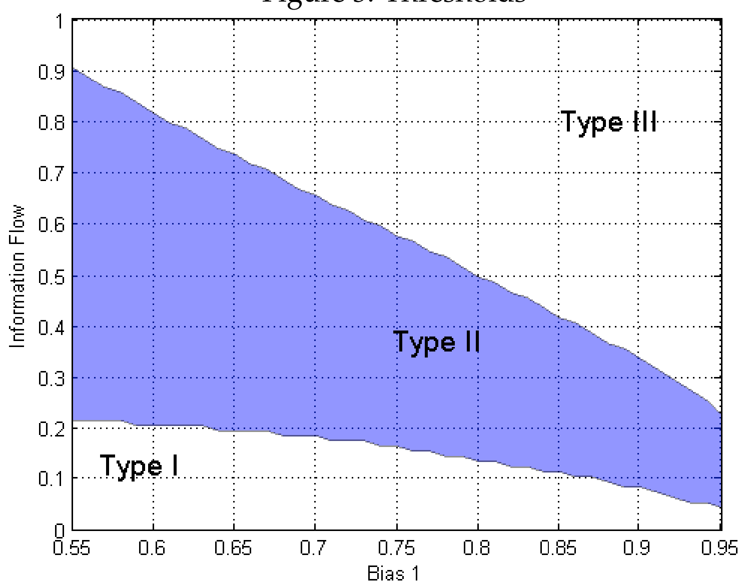
Thus, the probabilities assigned to the biased politician and the politician with the greatest distance to the biased one are opposite. This also implies that there is a continuum of equilibria due to the fact that nodes 2 and 4 are perfectly equivalent.

In the second type of equilibrium, L_1 assigns positive probability to all nodes, except to the one in his favor. L_2 will lobby the opposed politician or the politician, who has the greatest distance to the biased node.

Last, in equilibrium type 3, both lobbyists connect to the node opposite of the biased node. The Nash equilibrium is given by $s^{1*} = s^{2*} = s_3$.

These equilibrium strategies can be seen from Figure (2). I fix the ideological bias of politician 1 such that $\varphi_1 = .75$. The information flow varies. For each information flow, I depict the probability with which a politician is chosen. The strategies are depicted such that if there is a range in which the probabilities can lie, the median is given. Therefore the equilibria shown might not be unique. However, if we observe zero probability assigned to a politician, then this node will never be chosen in equilibrium.

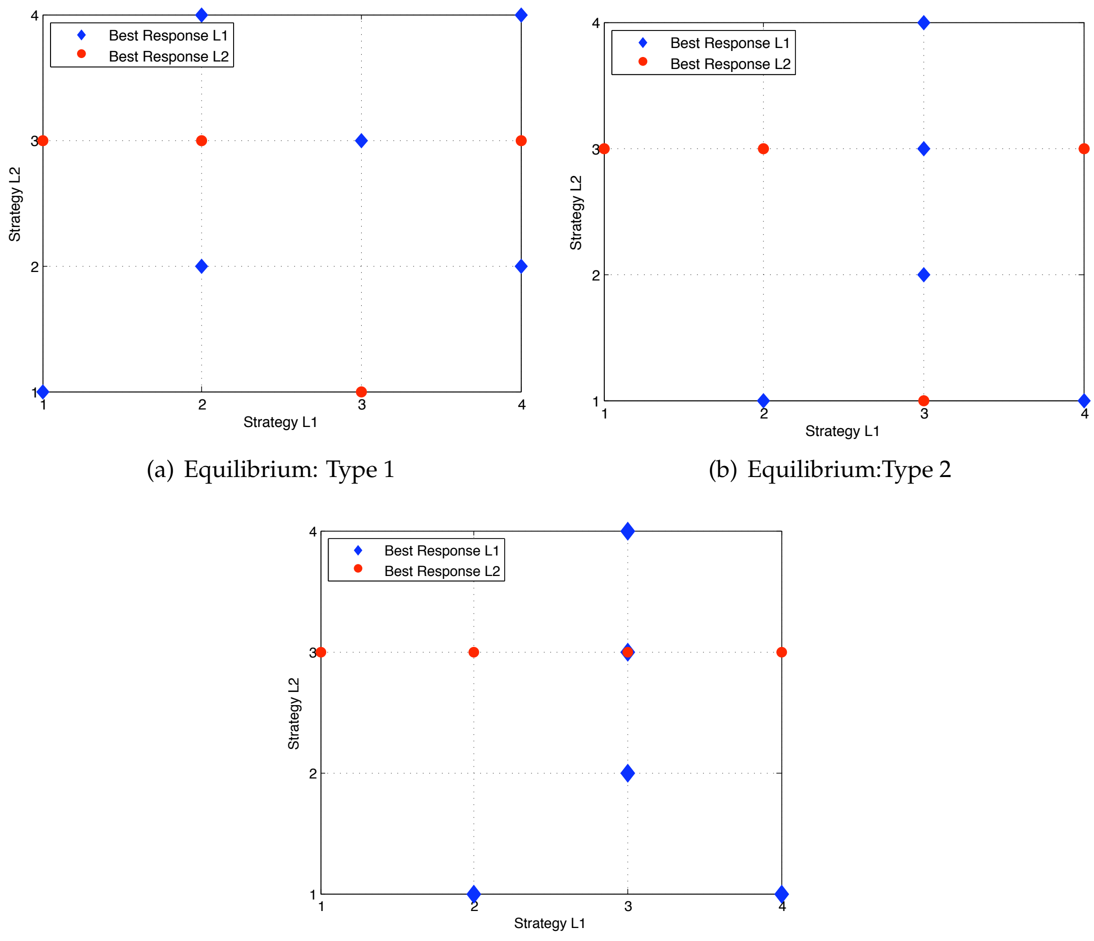
Figure 3: Thresholds



The question is, when which equilibrium occurs. This can be seen from Figure (3), which shows that the magnitude of the information flow has a greater influence on the type of equilibrium to emerge than the level of the bias. For each bias, all three types of equilibria are possible. This is not true for the information flow. A very high or very low information flow determines the type of equilibrium independently of the bias. Only for intermediate information flows, does the bias matter. But even then, only two types of equilibria can arise.

Thus, when the information flow is high, then we always find the symmetric Nash equilibrium, where both lobbyists choose the unbiased politician with the greatest distance to the biased one. This equilibrium also arises, when the information flow is intermediate and the bias is high. If both the bias and the information flow are intermediate, then the type 2 equilibrium emerges. Otherwise, when the information flow is low, we find that the equilibrium involves mixing between all politicians.

Figure 4: Best Responses



(a) Equilibrium: Type 1

(b) Equilibrium:Type 2

(c) Equilibrium:Type 3

Figure (4) , which depicts the best response functions, helps to understand the intuition behind the three different types of equilibria. The blue diamonds give the best response of lobbyist 1, the red dots the best response of L_2. As mentioned before, nodes 2 and 4 are perfectly equivalent. Thus, if node 2 is a best response, so is node 4 and vice versa.

The intuition for the type 1 equilibrium here is that the L_1 prefers to be at the same node as L_2, whereas L_2 prefers to be at a different one. Therefore, L_2 tries to get away from L_1, resulting in mixing between all politicians. Note that in a Nash equilibrium in pure strategies, the players do not necessarily only mix between the strategies that are a best response to a pure strategy. Therefore, one can observe something like here where there is mixing between all strategies although for L_2 choosing politician 2 or 4 is never a best response.

Both in the type 1 and 2 equilibrium, it is either a best response for L_2 to choose node 1 or 3. He prefers node 3 whenever L_1 is not connected to politician 3. If, however, L_1 chooses node 3, then L_2 prefers to connect to the opposing politician as the distance to L_1 is maximized and he can somewhat neutralize the bias of politician 1. But L_1's best response function changes from the type 1 equilibrium to the type 2 equilibrium. As the information flow or the bias are larger, L_1 does not want to defend the politician biased in his favor anymore, but only chooses an unbiased neighbor of the biased politician. Also, he does not want to be at the same node as L_2 any more, but prefers to lobby node 3 directly, whenever L_2 chooses one of the unbiased politicians.

If the information flow or the bias is high, as is the case in the type 3 equilibrium, then the biased politician will vote for the cause of L_1 with a probability of more than $\frac{1}{2}$, independently of the strategies of the lobbyists. Therefore, L_2 tries to get as far away from politician 1 as possible, as this politician is already lost to him. L_1 has no incentive to connect to politician 1 as he has already won him over. Therefore, it is also better for him to lobby the unbiased political actor, with the greatest distance to the biased politician, as this politician has also an unbiased neighborhood. This implies that the effect of ideology spreads through the network. An unbiased politician, who is close to a biased one will not be lobbied due to his neighborhood.

Thus, when the information flow is low, then it is most important for L_1 to be at the same node as L_2, whereas it is most important for L_2 to be at a different node. This leads to the mixing between all the strategies. But as the information flow or the bias increases, being at the same or different node is of less importance. The biased politician is less likely to be convinced and the lobbyists focus on the unbiased politicians.

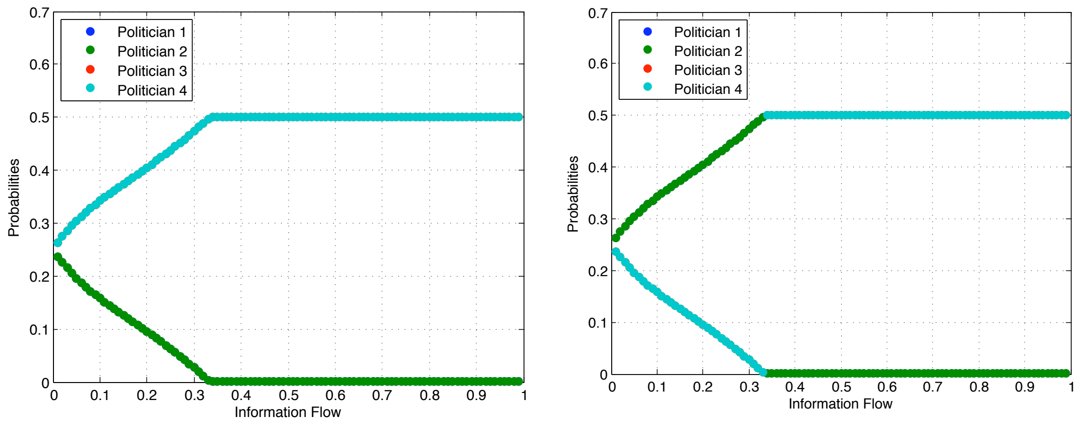
Ring: 2 biased and 2 neutral politicians

In this setting the both biased politicians are in favor of L_1. In the ring there are now two possible constellations. The biased agents can be next to each other or opposite.

Suppose first that the biased politicians are direct neighbours. Let w.l.o.g. politician 1 and 2 be the biased politicians. There are two types of Nash Equilibria. In the first type of equilibrium, both lobbyists assign positive probability to all politicians, whereas in the second type, only the unbiased political actors are lobbied.

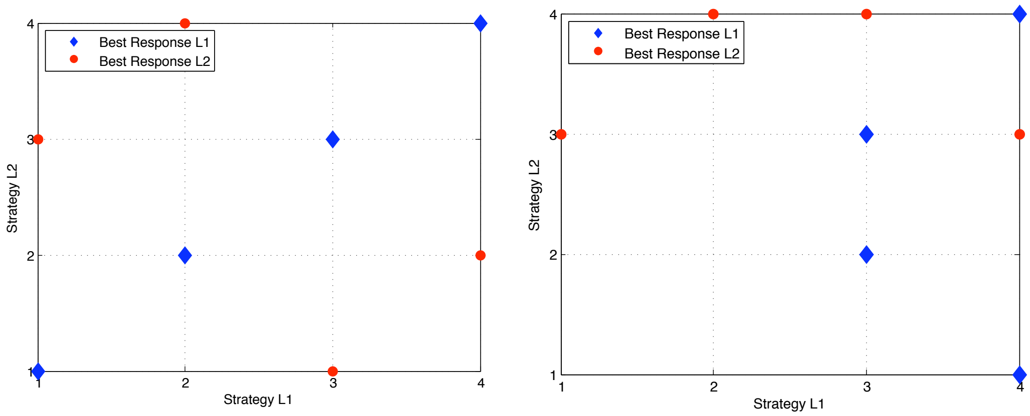
The equilibrium strategies can be seen in Figure (5). The probabilities assigned to politicians 1 and 2 are the same as are the probabilities assigned to politicians 3 and 4. Here, unlike in the previous case, the equilibria are unique. There are no two nodes that are equivalent anymore. That this is true can also seen from the best response function, which is given in Figure (6).

Figure 5: Equilibrium Strategies: 2 Biased and 2 Unbiased Politicians (Neighbors) ($\varphi_1 = \frac{3}{4}$)



(a) Equilibrium Strategy of L_1 (b) Equilibrium Strategy of L_2

Figure 6: Best Response Functions: 2 Biased and 2 Unbiased Politicians (Neighbors)



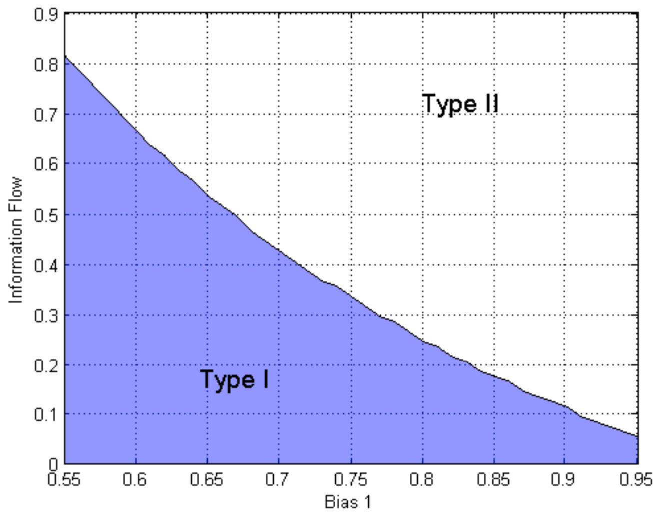
(a) Type 1 Equilibrium (b) Type 2 Equilibrium

In the first type of equilibrium, L_1 prefers to be at the same node as L_2, whereas L_2 prefers to lobby the politician who has the greatest distance to the politician chosen by L_1, independent of the bias of the politicians. It only matters to get as far away from L_1 as possible. In the second equilibrium, both lobbyists find it optimal to choose only the unbiased politicians. But as L_1 prefers to be at the same node as L_2 and L_2 prefers to be at a different node than L_1, there cannot be a Nash equilibrium in pure strategies.

It remains to be shown when which type of equilibrium occurs. This is easiest to see in Figure (7). Again, the information flow reinforces the bias. If both the bias and the information flow are relatively low, then we have mixing between all politicians and if the bias and the information flow are high, only the unbiased politicians are lobbied with positive probability.

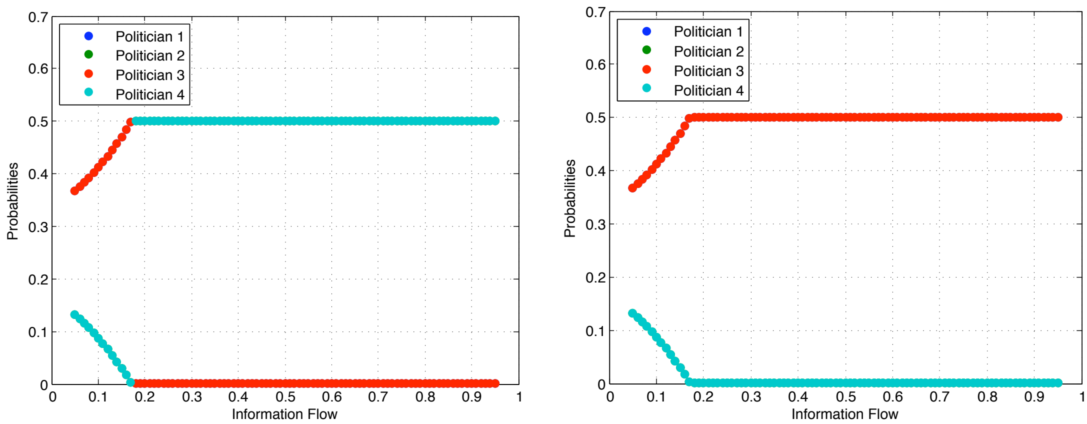
The intuition here is similar to the intuition where there is only one biased politician and three unbiased ones. As the information flow and the bias are high, the decision of

Figure 7: Threshold: 2 Biased and 2 Unbiased Politicians (Neighbors)



the biased politicians cannot be sufficiently changed as to make the biased politicians a valuable target. This is different when the information flow and the bias are low. Then, the opinion of the biased politicians can still be affected.

Figure 8: Equilibrium Strategies: 2 Biased and 2 Unbiased Politicians (Opposite), $\varphi_1 = \frac{3}{4}$



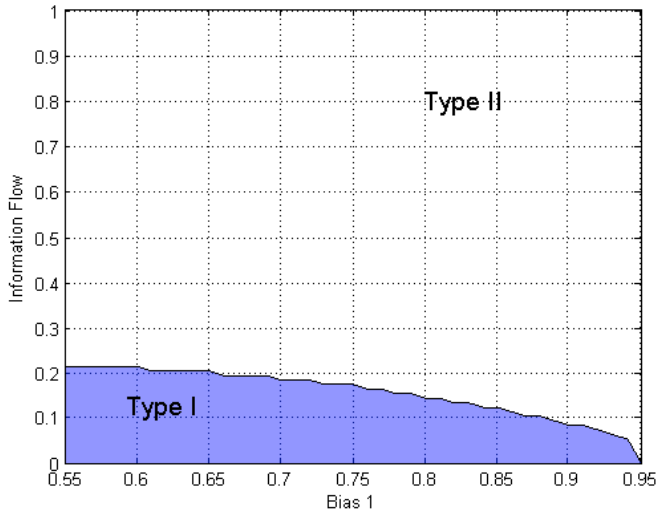
(a) Equilibrium Strategy of L_1

(b) Equilibrium Strategy of L_2

Now, consider the case where the biased politicians are opposite of each other. Assume w.l.o.g. that the biased politicians are politicians 1 and 3. Note that in this case politicians 2 and 4 are equivalent and so the equilibrium strategies given in Figure (8) are not unique strategies. There are again two kinds of equilibria. In the type 1 equilibrium, there is mixing between all politicians, in the type 2 equilibrium, L_1 mixes between the unbiased politicians, whereas lobbyist 2 mixes between the two biased political actors.

Again, which type of equilibrium occurs depends on the information flow and the bias. If the information flow is low, then the type 1 equilibrium occurs. Note here the difference between the size of the regions when the biased politicians are next to each

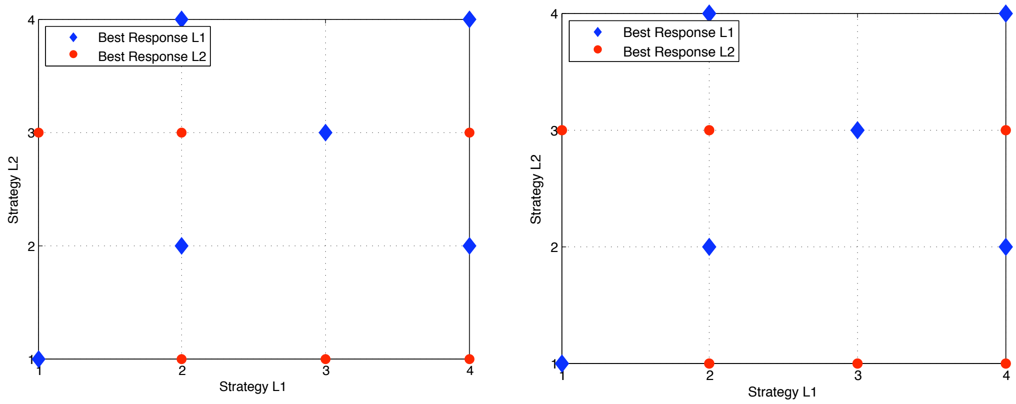
Figure 9: Threshold: 2 Biased and 2 Unbiased Politicians (Opposite)



other and opposite of each other. If the biased politicians are direct neighbors, then it is more likely to have mixing between all agents, than when they are opposite of each other.

To gain some intuition for the equilibria, consider the best responses in Figure (10). As one can see the best response functions do not change. However, the payoffs change

Figure 10: Best Response Functions: 2 Biased and 2 Unbiased Politicians (Opposite)



(a) Type 1 Equilibrium

(b) Type 2 Equilibrium

such that it becomes better to only lobby the unbiased politicians for L_1, instead of trying to chase L_2 as can be seen from the equilibrium strategies given in Figure (8). L_1 always prefers to be at the same node as L_2. L_2 always chooses one of the biased politicians. He is indifferent between the biased ones, when L_1 chooses an unbiased political actor. If L_1 chooses a biased politician, L_2 chooses the other biased one.

As can be seen there are significant differences between the equilibrium strategies when the biased politicians are next to each other and when they are opposite of each other. Consider first the equilibrium where there is mixing between all politicians. When politicians with the same ideological bias are next to each other then, the probabilities assigned to the biased versus the unbiased politicians are opposite, in the sense that

$$\sigma^1(s_1) = \sigma^1(s_2) = \sigma^2(s_3) = \sigma^2(s_4)$$
$$\sigma^1(s_3) = \sigma^1(s_4) = \sigma^2(s_1) = \sigma^2(s_2).$$

If the ideological politicians are not neighbors, but opposite of each other, then the probabilities assigned are the same,

$$\sigma^1(s_1) = \sigma^1(s_3) = \sigma^2(s_1) = \sigma^2(s_3)$$
$$\sigma^1(s_2) + \sigma^1(s_4) = \sigma^2(s_2) + \sigma^2(s_4).$$

The probabilities are the same due to the additional symmetry of the politicians on the ring, which leads to a symmetric payoff matrix for both lobbyists. This does not hold when the biased politicians are neighbors. Then, the probabilities assigned have to be opposite due to the structure of the payoff matrix.

But more importantly, when the biased politicians are neighbors and both the information flow and the bias are sufficiently large, then the structure of the equilibria differs completely. Unlike in the case where the biased politicians are neighbors, the equilibrium if the biased political actors are opposite of each other is asymmetric and positive probability is not only assigned to the unbiased politicians. Here, L_2 has an incentive to lobby a biased politician as this implies that he only has unbiased neighbors. This is better than lobbying an unbiased politician with two biased neighbors. Further, L_1 does not have an incentive to choose one of the biased politicians. If he chose a biased politician, he would be with some probability at the same node as L_2, but with some probability the distance between them would be two. This is worse than always being at a distance of one due to the concavity of $\frac{\varphi_1}{\varphi_1+\delta^i(1-\varphi_1)}$, where $i \in \{-2,-1,0,1,2\}$ when the information flow is sufficiently high. To be more specific,

$$\frac{\delta\varphi_1}{\delta\varphi_1+1-\varphi_1} + \frac{\varphi_1}{\varphi_1+\delta(1-\varphi_1)} > \varphi_1 + \frac{1}{2}\left(\frac{\delta^2\varphi_1}{\delta^2\varphi_1+1-\varphi_1} + \frac{\varphi_1}{\varphi_1+\delta^2(1-\varphi_1)}\right),$$

whenever $\delta > \overline{\delta}$.[6] That the strategic considerations differ depending on whether the biased politicians form a block can also be seen from the best response functions. When the biased politicians are opposite of each other, then L_2 always prefers to choose the biased one. In the case where the biased nodes are next to each other, he only cares about maximizing the distance to the politician L_1 chose, when the bias and the information flow are low. When the bias and the information flow are high, he is only interested in the unbiased politicians. L_1 still prefers to be at the same node as L_2, unless the biased nodes are neighbors and the bias and the information flow are high. In this case, the biased

[6] Solving for δ yields the somewhat complicated threshold. As the exact threshold does not give any insight, it is omitted here.

nodes can be disregarded as their opinion cannot be changed anymore.

Star In the star, in almost all cases both lobbyists connect to the center. There are some exceptions. If the bias of the center politician is very high, the information flow is high and there exists exactly one node on the periphery which is not as biased as the center, then both lobbyists connect to this less biased node on the periphery. A lobbyist only has an incentive to not lobby the center, when the center and the other nodes on the periphery are so biased that it is not really possible to change the outcome much. Then it is better to focus on the one node where it is still possible to change the opinion. But if there are more than two nodes on the periphery that are not much biased relative to the other nodes, then both lobbyists connect to the center. This result is hardly surprising. It shows that if a politician is very central, compared to the other political actors, he is a very attractive lobbying target.

4.2.1 A biased politician among unbiased ones

Suppose now, that there is one politician among all the political actors that is biased and assume without loss of generality that he is biased in favor of L_1.

Consider now a ring with n political actors. Let the node that is biased be node 1.

Lemma 2 (Ring). *Let n be even. Then the action space for each player can be reduced to*

$$S^{re} = \{s_1, s_2, \dots, s_{1+n/2}\}.$$

If n is odd, then the action space can be reduced to

$$S^{ro} = \{s_1, s_2, \dots, s_{(n+1)/2}\}.$$

Proof. Consider first the case when n is even.

Then, s_2 is payoff equivalent to s_n, s_3 is payoff equivalent to s_{n-1}, etc. Thus, the nodes that are equivalent have the same distance to node 1 as well as to $s_{1+\frac{n}{2}}$. Thus, we can define s_k^+ as the node that lies to the right of node 1 and is k links away from node 1. The nodes to the left are denoted by s_k^-. Note that $k \in \{2, \dots, \frac{n}{2} - 1\}$

Suppose that L_1 chooses some pure strategy s^1. Then the payoff of L_2 at nodes 1 and $s_{1+n/2}$ is the same for the equivalent nodes by definition. To be more precise,

$$p^1(s^1, s_k^+|s_1) = p^1(s^1, s_k^-|s_1)$$
$$p^1(s^1, s_k^+|s_{1+\frac{n}{2}}) = p^1(s^1, s_k^-|s_{1+\frac{n}{2}})$$

$\forall k$. As we have a constant sum game, the payoffs for L_2 are also equivalent.

Suppose now that L_1 connects to node 1. Then, clearly it does not matter whether L_2 chooses s_k^+ or s_k^-. The same is true when L_1 chooses node $1 + \frac{n}{2}$. Last, suppose that L_1 connects to a node, which is neither 1 nor the node furthest away from node 1. Again, L_2 is indifferent between s_k^+ and s_k^-, $\forall k$. The distance to node 1 and s_{1+n} is the same as mentioned before and so the relative distance to these nodes does not change. Therefore, the payoff at node 1 and $s_{1+n/2}$ is the same, whether L_2 chooses s_k^+ or s_k^-. Further, the average payoff at every unbiased node for which an equivalent node exists is $\frac{1}{2}$. This is obvious when they connect to the same node, but it also holds when they connect to different nodes. The intuition is the same as in the ring with only unbiased lobbyists. Due to the symmetry in the ring, for each node where lobbyist 1 has an advantage there exists a node where lobbyist 2 has the same advantage leading to the average of $\frac{1}{2}$. To be more precise, let L_1 choose node s^1, $s^1 \neq s_1, s_{1+\frac{n}{2}}$. L_2 is then indifferent between s_k^+ and s_k^-. To see this, note that the payoffs for lobbyist 1 are given by

$$\pi^1(s^1, s_k^+) = \frac{1}{n}(\frac{1}{2}(n-2) + \frac{\delta^{l(s^1,s_1)}\varphi_1}{\delta^{l(s^1,s_1)}\varphi_1 + \delta^{l(s_k^+,s_1)}(1-\varphi_1)} + \frac{\delta^{l(s_k^+,s_1)}}{\delta^{l(s_k^+,s_1)} + \delta^{l(s^1,s_1)}})$$
$$\pi^1(s^1, s_k^-) = \frac{1}{n}(\frac{1}{2}(n-2) + \frac{\delta^{l(s^1,s_1)}\varphi_1}{\delta^{l(s^1,s_1)}\varphi_1 + \delta^{l(s_k^-,s_1)}(1-\varphi_1)} + \frac{\delta^{l(s_k^-,s_1)}}{\delta^{l(s_k^-,s_1)} + \delta^{l(s^1,s_1)}})$$

As $l(s_k^+, s_1) = l(s_k^-, s_1)$, the payoffs have to be equivalent. Therefore, it suffices to restrict attention to the set S^{re}.

The argument when n is odd is similar. The only difference is now that every node except for node 1 has a node that is equivalent. But the argument carries through as before. □

What this lemma shows is that only the relative distances to the biased node matter. Therefore, we can rewrite the problem to make it more tractable. Let d be the number of links from node 1 to the node that has the greatest distance to this node. When n is even $d = \frac{n}{2}$, if n is odd, then the maximum distance is $\frac{n-1}{2}$.

Proposition 5 (Ring). *Let $\delta > (\frac{1-\varphi_1}{\varphi_1})^{\frac{1}{d}}$.Then the unique Nash equilibrium when n is even is defined by*

$$\sigma^1(s_{1+n/2}) = \sigma^2(s_{1+\frac{n}{2}}) = 1.$$

If n is odd the set of Nash equilibria is given by

$$\sigma^j(s_{1+\frac{n-1}{2}}) + \sigma^j(s_{2+\frac{n-1}{2}}) = 1, \quad j \in \{1, 2\}.$$

Proof. I consider first the case where n is even. Note that independently of the chosen strategies of both strategies every lobbyist has a share of $\frac{1}{2}(n-2)$ for sure. This is straightforward when they connect to the same node. It can be easily verified, when one of the lobbyist chooses either s_1 or $s_{1+\frac{n}{2}}$. It also holds when both lobbyists connect to different nodes that are neither s_1 nor $s_{1+\frac{n}{2}}$, as has been shown in the proof of Lemma 2.

The payoff thus only depends on two nodes. All possible payoffs for L_1 are given by:

$$\{\frac{\delta^d\varphi_1}{\delta^d\varphi_1 + 1 - \varphi_1} + \frac{1}{1+\delta^d}, \ldots, \varphi_1 + \frac{1}{2}, \ldots, \frac{\varphi_1}{\varphi_1 + \delta^d(1-\varphi_1)} + \frac{\delta^d}{1+\delta^d}\}$$

It holds that

$$\frac{\delta^i\varphi_1}{\delta^i\varphi_1+1-\varphi_1}+\frac{1}{1+\delta^i}>\varphi_1+\frac{1}{2}$$
$$>\frac{\varphi_1}{\varphi_1+\delta^j(1-\varphi_1)}+\frac{\delta^j}{1+\delta^j}\quad\forall i,j\in\{1,\ldots,d\},$$

for $\delta>(\frac{1-\varphi_1}{\varphi_1})^{\frac{1}{d}}$. This implies that L_1 prefers to be as far away from node the biased node as possible. The argument is the same for L_2. For this lobbyist, the different possible payoffs are

$$\{\frac{\delta^d(1-\varphi_1)}{\varphi_1+\delta^d(1-\varphi_1)}+\frac{1}{1+\delta^d},\ldots,(1-\varphi_1)+\frac{1}{2},\ldots,\frac{1-\varphi_1}{\delta^d\varphi_1+1-\varphi_1}+\frac{\delta^d}{1+\delta^d}\}.$$

Now,

$$\frac{\delta^i(1-\varphi_1)}{\varphi_1+\delta^i(1-\varphi_1)}+\frac{1}{1+\delta^i}>(1-\varphi_1)+\frac{1}{2}$$
$$>\frac{1-\varphi_1}{\delta^j\varphi_1+1-\varphi_1}+\frac{\delta^j}{1+\delta^j}\quad\forall i,j\in\{1,\ldots,d\},$$

when $\delta>(\frac{1-\varphi_1}{\varphi_1})^{\frac{1}{d}}$. This implies that L_2 would like to be further away from the biased node than L_1. Therefore in equilibrium, both choose to connect to node $1+\frac{n}{2}$. The argument is similar when n is odd. From Lemma 2 I know that only the distance to node 1 matters. As there are two nodes with the same distance, the lobbyists are indifferent between them. □

If the information flow is high, a politician who is close to a biased politician will not be lobbied, even if he is not biased himself.

Note that in Proposition 5, the restriction can also be placed on the bias. Thus, both lobbyists connect to the most distant node $\forall\delta$ and $\frac{1}{2}<\varphi_1<\frac{1}{1+\delta^d}$. This shows that there is a trade off between the bias and the information flow. If the bias is low, then the information flow has to be high to attain a Nash equilibrium where both connect to the politician who has the greatest distance to the biased one.

Proposition 6 (Ring). *Let $\delta<(\frac{1-\varphi_1}{\varphi_1})^{\frac{1}{d}}$. Then there does not exist a pure strategy Nash equilibrium.*

Proof. As before we can use the fact that for every strategy combination the payoff $\frac{1}{2}(n-2)$ is certain. Thus we can again restrict attention to the following sets of payoffs for L_1 and L_2 :

$$\{\frac{\delta^d\varphi_1}{\delta^d\varphi_1+1-\varphi_1}+\frac{1}{1+\delta^d},\ldots,\varphi_1+\frac{1}{2},\ldots,\frac{\varphi_1}{\varphi_1+\delta^d(1-\varphi_1)}+\frac{\delta^d}{1+\delta^d}\},$$
$$\{\frac{\delta^d(1-\varphi_1)}{\varphi_1+\delta^d(1-\varphi_1)}+\frac{1}{1+\delta^d},\ldots,(1-\varphi_1)+\frac{1}{2},\ldots,\frac{1-\varphi_1}{\delta^d\varphi_1+1-\varphi_1}+\frac{\delta^d}{1+\delta^d}\}.$$

We find that

$$\varphi_1 + \frac{1}{2} > \frac{\varphi_1}{\varphi_1 + \delta^i(1-\varphi_1)} + \frac{\delta^i}{1+\delta^i} \quad \forall i \in \{-d, \dots, d\}$$
$$(1-\varphi_1) + \frac{1}{2} < \frac{\delta^i(1-\varphi_1)}{\varphi_1 + \delta^i(1-\varphi_1)} + \frac{1}{1+\delta^i} \quad \forall i \in \{-d, \dots, d\}$$

This implies that L_1 wants to be at the same node as L_2, whereas L_2 wants to be at a different node and so there cannot be a Nash equilibrium in pure strategies. [7] □

Proposition 7 (Ring). *There exists $\delta < \overline{\delta}(\varphi_1)$, such that both lobbyists mix between all strategies.*

Proof. Suppose L_1 mixes between all strategies. For this to be the case it has to hold that

$$\frac{1}{2}\underbrace{\begin{pmatrix} \varphi_1 + \frac{1}{2} & \frac{\varphi_1}{\varphi_1+\delta(1-\varphi_1)} + \frac{\delta}{1+\delta} & \cdots & \frac{\varphi_1}{\varphi_1+\delta^d(1-\varphi_1)} + \frac{\delta^d}{1+\delta^d} \\ \frac{\delta\varphi_1}{\delta\varphi_1+(1-\varphi_1)} + \frac{1}{1+\delta} & \varphi_1 + \frac{1}{2} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\delta^d\varphi_1}{\delta^d\varphi_1+(1-\varphi_1)} + \frac{1}{1+\delta} & \cdots & \cdots & \varphi_1 + \frac{1}{2} \end{pmatrix}}_{=A} \begin{pmatrix} \sigma^2(s_1) \\ \sigma^2(s_2) \\ \vdots \\ \sigma^2(s_{1+d}) \end{pmatrix} = \underbrace{\begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_d \end{pmatrix}}_{=c},$$

where c denotes the payoffs. Now, let $\delta \to 0$.

$$\lim_{\delta \to 0} A = \begin{pmatrix} \varphi_1 + \frac{1}{2} & 1 & \cdots & 1 \\ 1 & \varphi_1 + \frac{1}{2} & \cdots & 1 \\ \vdots & \vdots & \ddots & 1 \\ 1 & 1 & \cdots & \varphi_1 + \frac{1}{2} \end{pmatrix}$$

Therefore, at the limit, to make L_1 indifferent between all the strategies, L_2 assigns equal probability to all the politicians. As the payoffs of L_2 in the limit are given by $B = 2 - A$, for L_2 to be indifferent, L_1 also has to mix between all his strategies with the same probabilities. Therefore,

$$\sigma^1(s_1) = \sigma^1(s_2) = \cdots = \sigma^1(s_d) = \sigma^2(s_1) = \sigma^2(s_2) = \cdots = \sigma^2(s_d) \equiv p$$

Now, does there exist a profitable deviation? The payoff of L_1 when mixing between all strategies is

$$dp^2(\varphi_1 + \frac{1}{2}) + (1 - dp^2).$$

But due to the structure of the payoff matrix, any strategy yield the same payoff. Therefore L_1 does not have an incentive to deviate. Due to the fact that we have a zero sum

[7] It also cannot be the case that L_2 plays a pure strategy in a Nash equilibrium, but it might be that L_1 plays a pure strategy and L_2 mixes between s_1 and s_{1+d}.

game, L_2 then also has no incentive to deviate. This shows that mixing between all strategies is indeed an equilibrium in the limit.

But as

$$\frac{\delta^i \varphi_1}{\delta^i \varphi_1 + (1-\varphi_1)} + \frac{1}{1+\delta^i} \qquad \forall i \in \{-d, \ldots, d\}$$

is continuous in δ and the overall payoff is a sum of continuous functions, there exists a $\overline{\delta}(\varphi_1)$ such that for $\delta < \overline{\delta}(\varphi_1)$ both lobbyists mix between all strategies. □

Now suppose that there are n_1 biased politicians and the structure of the network is a star.

Proposition 8 (Star). *Let $2n_1 \leq n$. Then both lobbyists connect to the center, independent of the position of the biased politician in the network, the actual bias and the information flow.*

Proof. First assume that a biased politician is in the center and denote this politician as politician 1. I will show first that it is indeed an equilibrium to connect to the central politician and then that this is the unique NE.

If both connect to the center, then the payoff of lobbyist 1 is

$$\Pi^1(s_1, s_1) = \frac{1}{n}(n_1\varphi_1 + \frac{1}{2}(n - n_1)).$$

If lobbyist 2 deviates to an unbiased node on the periphery, then the payoff of lobbyist 1 is

$$\Pi^1(s_1, s_{ub}) = \frac{1}{n}(n_1 \frac{\varphi_1}{\varphi_1 + \delta(1-\varphi_1)} + \frac{1}{1+\delta}(n - n_1 - 1) + \frac{\delta}{1+\delta}).$$

If he deviates to a biased politician on the periphery L_1's payoff is

$$\Pi^1(s_1, s_b) = \frac{1}{n}((n_1 - 1)\frac{\varphi_1}{\varphi_1 + \delta(1-\varphi_1)} + \frac{1}{1+\delta}(n - n_1) + \frac{\delta\varphi_1}{\delta\varphi_1 + (1-\varphi_1)}).$$

It can be easily verified that $\Pi^1(s_1, s_1)$ is lower than $\Pi^1(s_1, s_{ub})$ and $\Pi^1(s_1, s_b)$, which implies that it cannot be a profitable deviation for lobbyist 2 to connect to a node on the periphery, be it biased or unbiased. Now, suppose L_1 deviates, while L_2 remains connected to the center.

$$\Pi^1(s_{ub}, s_1) = \frac{1}{n}(\frac{1}{1+\delta} + (n - n_1 - 1)\frac{\delta}{1+\delta} + n_1 \frac{\delta\varphi_1}{\delta\varphi_1 + 1 - \varphi_1})$$
$$\Pi^1(s_b, s_1) = \frac{1}{n}(\frac{\varphi_1}{\varphi_1 + \delta(1-\varphi_1)} + (n - n_1)\frac{\delta}{1+\delta} + (n_1 - 1)\frac{\delta\varphi_1}{\delta\varphi_1 + 1 - \varphi_1})$$

Then both payoffs are lower than $\Pi^1(s_1, s_1)$, which confirms that it is indeed a NE if both connect to the center. It is also the unique NE as connecting to the center as for lobbyist 1 this is a strictly dominant strategy. We have already seen that it is the unique best response to connect to the center when L_2 connects to the center. It remains to be shown that it also is the unique best response when the other agent does not connect to the center.

$$\Pi^1(s_1, s_{ub}) = \frac{1}{n}(n_1 \frac{\varphi_1}{\varphi_1 + \delta(1-\varphi_1)} + \frac{1}{1+\delta}(n - n_1 - 1) + \frac{\delta}{1+\delta}) \tag{3}$$

$$\Pi^1(s_{ub}, s_{ub}) = \Pi_1(s_1, s_1) \tag{4}$$

$$\Pi^1(s_b, s_{ub}) = \frac{1}{n}((n_1 - 1)\varphi_1 + \frac{\varphi_1}{\varphi_1 + \delta(1-\varphi_1)} + \frac{\delta}{1+\delta} + \frac{1}{2}(n - n_1 - 1)) \tag{5}$$

It has already been shown that (3) is greater than (4) and it is straightforward to show that (3) is larger than (5).

$$\Pi^1(s_1, s_b) = \frac{1}{n}((n_1 - 1)\frac{\varphi_1}{\varphi_1 + \delta(1-\varphi_1)} + \frac{1}{1+\delta}(n - n_1) + \frac{\delta\varphi_1}{\delta\varphi_1 + (1-\varphi_1)}) \tag{6}$$

$$\Pi^1(s_{ub}, s_b) = \frac{1}{n}((n_1 - 1)\varphi_1 + \frac{\delta\varphi_1}{\delta\varphi_1 + (1-\varphi_1)} + \frac{1}{1+\delta} + \frac{1}{2}(n - n_1 - 1)) \tag{7}$$

$$\Pi^1(s_b, s_b) = \Pi_1(s_1, s_1) \tag{8}$$

Again, we know that $\Pi^1(s_1, s_b) > \Pi 1(s_b, s_b)$. Further, (6) is larger than (8). This proves that connecting to the center is a strictly dominant strategy. Therefore connecting to the center is the unique NE. □

This shows that the bias still does not matter for the chosen strategy, as long as there are not too many biased politicians.

5 Conclusion

Unsurprisingly, a politician, that is more central in the network is more likely to be lobbied than a politician who is not that central. Thus, in the star, it is very likely to find that the central politician is lobbied independently of the bias and information flow within the network. This does not hold if the politicians are the same in terms of centrality. There the magnitude of the bias as well as the strength of the ties matter for the optimal lobbying strategy. It has been shown for the example of four politicians that not only the magnitude of the bias and the information flow matter but also the position of the biased nodes, even if the centrality is the same for all nodes.

It can be shown generally that there always exists an equilibrium where both lobbyists mix between all the politicians as well as there is always an equilibrium where the most unbiased politician is lobbied, when the network has a ring structure. In the star, if only a minority of political actors is biased, then the center will always be the lobbying target.

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