

PRELIMINARY DRAFT

Are Democrats Better Off With A Smaller Majority In The Senate?

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Abstract: In the current US Senate, the majority usually needs 60 votes to pass a bill, for institutional reasons. The Democrats basically had 60 senators starting the 111th Congress in 2009. The election of Scott Brown in February 2010 to fill the seat of late-Senator Ted Kennedy lowered this number to 59 members, and started a debate on whether Democrats would be able to pass their bills, in particular health-care reform. After the midterm elections of 2010, Democrats are holding a 52 seats majority. The majority needs to reach across the aisle to pass legislation. I argue, using a theoretical model, that the majority party might be able to attract more senators from across the aisle when they have a smaller majority. The simple idea is that the majority is trying to attract centrist opponents from a larger pool, and that mass defection might be less costly for a minority member than a lonely one.

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1 Question and Motivation

1.1 Introduction

Democrats needed a super-majority in the Senate in order to pass health-care reform. Though they were actually holding the 60 votes needed, there are two issues that seem to create a need for attracting members of the other party:

- Uncertainty about ‘Blue Dogs’ Democrats’ position on, say, abortion or other issues, might make it less costly to attract some liberal Republicans than some conservative Democrats;
- Bipartisanship might have a value *per se* for the majority party.

The media has heavily reported on the concessions given to the two Senators of Maine Olympia Snowe and Susan Collins in order to get their votes on the bill drafted by the Senate Finance Committee . Politico writes on September, 4th, while Max Baucus’ ‘Gang of Six’ was still discussing the legislation:

If the six-way talks break down, Democrats appear to have at least two major options left for passing a health care overhaul this year, sources say. The first option is to write a bill that could garner 60 votes, and pass through regular order. This legislation would hinge on support from Sen. Olympia Snowe (R-Maine), and perhaps Sen. Susan Collins (R-Maine) (...). The White House is in the midst of discussions with Snowe on a scaled-back bill that would not create a government-run insurance plan on day one(...) [but would include a] ‘trigger’ mechanism. (...) The ‘trigger’ approach would appeal to conservative Democrats and moderate Republicans(...)

The ‘trigger’ approach was, indeed, a favorite idea of Senator Snowe. At this point, attracting Republicans seemed already costly. The Health Care debate, as we know, is still not resolved and the Democratic plan(s) is(are) not attracting Republican support. Finally, the upheaval in Massachusetts with the election of Scott Brown to take the seat of late Senator Kennedy seemed to have annihilated all hopes for Democrats of a bipartisan health-care bill.

However, one interesting fact occurred in the week of February, 22nd. Scott Brown and four other Republicans, including the two Maine senators, decided to break ranks and not filibuster a \$15 billion ‘jobs bill’ supported by Harry Reid, the Democratic Senate Floor Leader. They also ended up voting for the bill when it was brought to the floor.

1.2 My main theory and its justification

In this paper, I will show that an increase in the number of members from the minority(at this day, the Republicans) does not weaken the ability of the majority to pass laws, and might actually even increase

their probability of success. The reason for this counter-intuitive effect is that the minority party cannot sanction those who defect harshly when there are multiple defections. To model this idea, we will assume that the party has a fixed amount of ‘punishment’ and spreads it equally between defectors. I also discuss potential responsibility diffusion effects that might be a topic for future research based on this idea.

Before looking at the model and the literature, I have to defend several point.

First, I consider a group of centrist Republicans, and I look at decisions made at the individual level. The reason why I think this is a good framework is that at the centre of the political spectrum, there seems to be a conflict between the party and the individual. On the Republican Side, Olympia Snowe voted with the party two-third of the time, so did Susan Collins. Scott Brown voted 82% of the time with Republicans. The next percentages are above 9 in 10. On the Democratic side, Ben Nelson voted with the party 67.6% of the time, interestingly enough, this number is lower than for Joe Lieberman(90.6%). I consider that those individuals, for ideological and electoral reasons, will like some bills from across the aisle and are willing to vote for it, *absent* any constraints. However, as I will argue below, they belong to a party and are playing a repeated game. This inherent conflict is what drives the results of this paper.

Second, I will often assume that the minority defections are more important than the majority defections. Basically, this assumption will mean that voters are *sure* about the number of defections needed for a majority bill to pass. I justify this assumption by looking, for instance, at the 110th Congress. Among the 20 Senators who voted with their party with the lowest frequency, 16 of them were Republicans.

Moreover, I argue that the potential sanction that a Republican suffers from defecting is actually higher than the benefits it gets from the Democrats by voting for their policy. There is indeed not much evidence in recent events that the Republicans got sweeteners from voting with the Democrats. Scott Brown or Olympia Snowe are believed to have voted ideologically on the bill, rather than having been attracted by self-interested benefits given by Democrats. More concrete examples can be taken from political analysis of other recent measures.

Recently, the Republicans started threatening to filibuster a financial reform bill but backtracked. Alexander Bolton(2010)[9] explains on April, 15th 2010 that Mitch McConnell, the Senate minority leader, had troubles maintaining cohesion in the party because a substantial fraction of Republicans wanted to ‘defect’. Here, the aim of McConnell was to block the discussion of the Democratic bill, while some Republicans Congressmen were reluctant to filibuster.

Senate Republican Leader Mitch McConnell (Ky.) does not have enough solid commitments from GOP lawmakers to block consideration of a Democratic Wall Street reform bill.

McConnell has circulated a letter within the GOP conference that would have Republicans pledge to block a motion to proceed to a financial regulatory reform bill unless Democrats

agree to reopen it for bipartisan negotiations.

But McConnell has fallen short of the 41 signatures he needs to send Senate Majority Leader Harry Reid (D-Nev.) a clear signal.

So far, Sen. Susan Collins (R-Maine), a crucial swing vote, has declined to sign the letter, according to a Republican source.

Ezra Klein(2010)[31] comments on the story and basically wrote the introduction to this paper. The interesting point is that a Republican will have more trouble to defect when he/she is alone. There are two interesting things in the comment below. First, political commentators talk explicitly about punishment, and the diffusion of punishment. Second, it is argued that the problem comes from the Republican Party's cohesion, and not by any sweet deals offered by the Democrats.

Right now, the only named dissenter is Sen. Susan Collins. Whether she's the only dissenter or one of a handful, however, is very important. One thing we've seen at various point in the past year is that it's very, very difficult for any Republican to be the only Republican on a major Democratic bill. Olympia Snowe can negotiate on health-care reform and Bob Corker can sit with Chris Dodd on financial reform, but at the end of the day, if they can't bring anyone onto the compromise they've hammered out, they probably can't commit to it, either.

It's one thing to be part of a moderate bloc bucking the party. Blame is diffuse, and so too is punishment. It's another thing to be the only Republican bucking the party. Then blame, and punishment, are concentrated. You'll get made an example of.

Matt Steinglass(2010)[44] of The Economist makes a more general comment on the fact that increasing the number of members of the minority shifts the pivotal power towards the minority. Then, the sunlight is on the minority's behavior(this is linked to Klein's comment on 'you will get made an example of'), so that it may be harder for the party to punish the defectors, and it might be easier for the centrist to defect(or harder to maintain the party line) if they want to satisfy their constituency. This will be captured by assuming that the punishment is inversely proportional to the number of potential defectors.

In early January, when Scott Brown won his election in Massachusetts and Democrats lost their 60-vote supermajority in the Senate, there was speculation that the silver lining for Democrats might be that media attention would shift away from potential Democratic defectors and towards potential Republican ones.

That, in fact, seems to be happening. There have been no highly publicised episodes of Democrats flamboyantly proclaiming their discomfort with this or that aspect of the various

financial-reform bills. Instead, we've had close coverage of Bob Corker's displeasure at his own leadership's misleading characterisations of a bill he helped negotiate. We've had rampant speculation about whether Mr Corker, Susan Collins, or Scott Brown might be persuaded to break with their caucus and push a bill past a GOP filibuster. We've had widespread coverage of reform-friendly pronouncements (and votes) by Charles Grassley and Richard Shelby. When Democrats had 60 votes, Democrats had decision power. Now that Republicans have 41 votes, Republicans have decision power.

More recently, the discussion of the repeal of the 'Don't Ask Don't Tell' policy justifies my question. As of November, 11th, 3 or 4 Republicans decided to vote with the Democrats on the military spending bill that includes the repeal. Lawrence O'Donnell(a former Democratic senator's staff director and one of the writer behind the West Wing), talking on MSNBC, specifically said that 'they are indicators of more to come(...) it won't be that one of those Republicans will have to then be blamed as being the one who delivered this bill'

One interesting fact is that we will have possibly another test of my theory in the near future. On November, 29th, Senator-elect Mark Kirk from Illinois is supposed to take the seat left vacant by President Barack Obama, changing the balance of power in the Senate from 59-11 to 58-12. The recent debate on the Bush tax cuts, the START treaty and the bill on military spending featuring a repeal of the Don't Ask Don't Tell policy should be an interesting test. However, we still have a couple of examples that can show the importance of those pivotal Republicans. In an post in September 2010 about campaign finance reform, Charles Schumer(D-NY) had 58 votes and was looking to attract Olympia Snowe, Susan Collins and Scott Brown, 'who say they support reform and disclosure in principle but objected to particular aspects of the bill(...)This, however, presumes the three would-be GOP reformers are willing to put their votes where their public declarations are, in the face of enormous pressure to side with the Consolidated Megacorps of the world.'

Finally, we can have a look at a hot debate in recent news. A lot of debate has happened on the 'filibuster', or the possibility to extend debate unlimitedly in the Senate with, usually, only 40 members(or $\frac{2}{5}$) of the entire chamber. Indeed, the rate of filibusters has dramatically increased in the past 30 years. My theory can provide another explanation. In Figure 1, one can see the number of cloture votes filed per months after 1973.

The two main reforms concerning the filibuster occurred in 1917 and 1975. In 1917, 'cloture' was put in the rules, to allow two-third of voting senators to close debate on a bill. In 1975, a new reform created the 60-votes Senate that we know today. In my theory, the fact that the majority needed to attract fewer Senators was not necessary a good thing for them. Indeed, it is possible that reducing the requiring number of defections from the minority has a negative impact on the probability of breaking

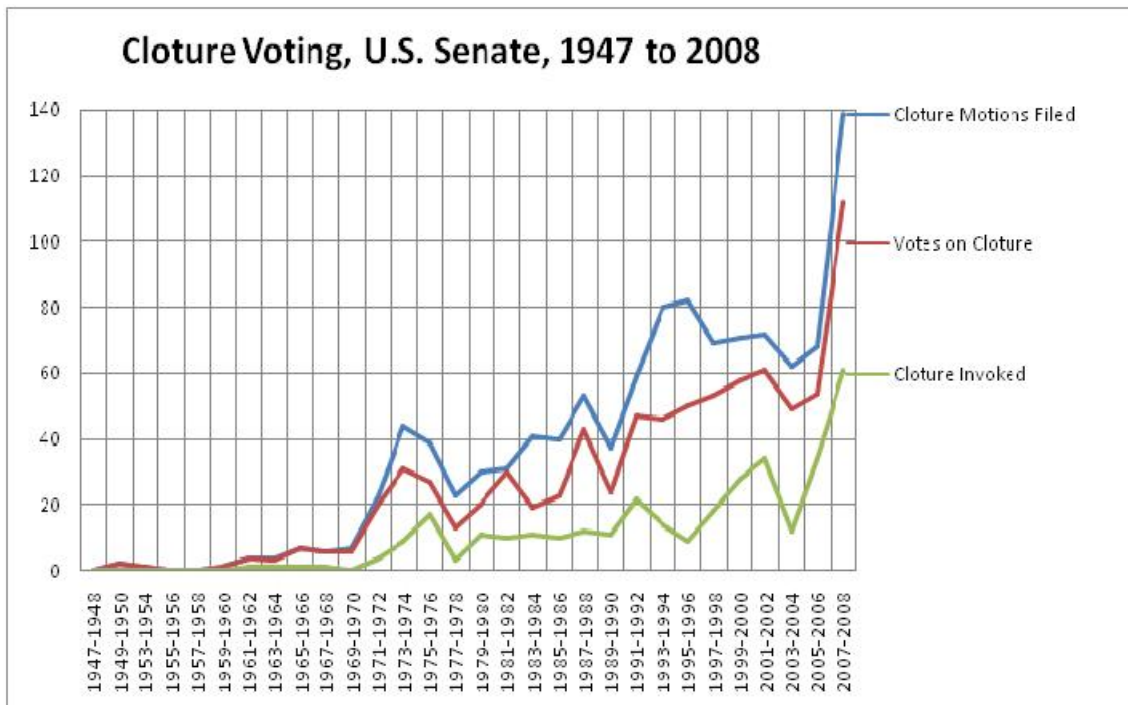


Figure 1: Number of cloture votes in the US Senate

the filibuster. This is the case if we assume that some members of the minority want some majority bill to pass but are constrained by the possible sanctions they might face from their party. Moreover, if this sanction is proportional to the number of defectors, in the sense that the party cannot sustain a high level of punishment for a lot of senators, the fact that more defectors are required might provide some more incentive to vote across the aisle.

1.3 Question and Hypothesis

This raises an interesting question: **is it actually easier for Democrats to reach some bipartisan consensus on bills when holding 59 votes than 60 votes?**

My hypothesis is two-fold.

- 1 A less numerous majority faces a more amenable minority, in the sense that the increase in the minority size is due to the election of centrists in marginal constituencies;
- 2 it is easier to fold a member of the opposite party if the probability that he/she will not be the only one to defect is high. I focus on this question by showing through which channel this might occur. I underline a material effect through a division of punishment effect, and a immaterial effect through some results from psychology.

2 Literature and Theories

2.1 Related Literature

To the best of my knowledge so far, the paper that is matching the issue I want to address the closest is Groseclose and Snyder(1996)[26].

In this paper, the authors show by some examples that the majority party often tries to attract more members to vote for bills that the minimal amount required to hold a majority and win a vote. They argue that along with uncertainty about some members, another reason motivates this behavior: if Republicans and Democrats are competing for marginal senators, by bribing more Republicans the Democrats make it more costly for Republicans to ‘bribe back’ in a sequential game. If only one Republican had been bribed, then it would have been easier/cheaper for the Republicans to counter the attack.

Thus, Groseclose et al.(1996) are looking at the group behavior, and at the behavior of the buyer. My proposal tackles the decision of the marginal senator who is being bought.

Thus, I am looking at one’s individual behavior. We want to find mechanisms through which an increase in the size of the minority can make a marginal senator’s defection more likely. I see two possible explanations

- The material consequence of defecting is lower when there are more defectors. If there are more defectors, the party can hardly be credible by punishing everybody for instance. It might also be costly to punish 4 senators the same way. Finally, it might be difficult for the party to assign the true distribution of responsibility in the defections. When can there be more defectors? My premise is that when the minority increases in size, it has to be because one more ‘centrist’ element has joined the rank(e.g. Scott Brown in Massachusetts). I will call those centrist senators ‘Brownites’.
- The psychological consequence of defecting is lower when there are more defectors. This is the responsibility diffusion theory, and can also be related to the ‘conformity’ literature.

One might also be interested in the interaction between those two channels.

2.2 Voting literature

This paper is also related to the literature on strategic voting, costly voting and public good provision.

Participation Game The game I will analyze is a participation game, as defined by Palfrey et al.(1983)[40]. In those kind of games, there is a motivation to vote in order to obtain the winners’ outcome. However, there is an incentive to free-ride and have people voting in your place, in order not to incur the cost of voting.

Strategic Voting The strategic and costly voting literature often assumes that there are two alternatives, and that decisions are majority-based with random tie-break. This is, for instance, the setting in the seminal articles of Palfrey et al.(1983, 1985)[40, 39]. In the first paper, they show that aggregate voting patterns, notably turnouts, could be understood through a strategic behavior of individuals when voting. Namely, the individuals condition their decision on whether they will be pivotal or not, i.e. whether they can change the outcome(or at least the probabilities of different outcomes). I am using a strategic view of voting in this paper, where the centrist Republicans are estimating the probability of different events to run a cost-benefit analysis of voting.

Costly Voting In Palfrey et al.(1983), the authors also introduce a completely symmetric framework where all individuals have identical cost of voting and where the payoffs are symmetric between the minority and the majority. The issue is that they find a large number of possible equilibria in pure and mixed strategies, so that it is quite uninformative. In the latter, they introduce heterogeneous cost of voting and show that only agents with low cost would turnout(when the electorate is large). In my paper, I introduce heterogeneous benefits from voting that will yield a similar intuition, along with a constant aggregate punishment but with uncertainty over the individual level.

The latter characteristic could be seen as a cost of voting. One difference with previous literature is that here, the cost of voting is uncertain to all players, there is no private information involved. The private information is at the level of the uncertain benefit.

Thus, there is an explicit cost of voting through the exogenous punishment. One extension here is that the punishment is divided among the voters, and the number of voters is endogenous. Moreover, as far as I know, the literature on costly voting is usually more general, and therefore less grounded on precise events, compared to what I am proposing to do here.

Pivotal events An interesting thing to observe in the context of the problem I am proposing to tackle is that the probability of being pivotal is substantial. In traditional costly voting model, with large electorate, the pivotality is simply looking at the probability of whether the voter can change the outcome from a defeat to a tie or from a tie to a victory. With a punishment that is spread among defectors, one centrist Republican has to condition on every possible number of defectors, since depending on how many defectors there is, the cost of voting will differ. The way we are going to solve the problem is similar to Palfrey et al.(1985), and will also look like Goeree et al.(2006)[23] and Levine et al.(2007)[34]. Namely, our analysis of the strategic equilibria with heterogeneous benefits will follow the same steps as Levine et al.(2007). In this paper of costly voting with a majority and a minority, the authors show that agents use cutoff strategies, and that the cutoff costs are determined endogeneously through the probability that they imply on pivotal events. My paper proceeds to a similar analysis. First, we will show that only

those who derive a benefit from bill above a certain threshold are going to vote for it. This threshold thus determines the probability that an individual will vote: it will be the ex-ante probability of having a benefit greater than the threshold. Hence, a voter will be able to know the probability of each number of defectors and condition his/her vote on this.

2.3 Psychological channels

In the following two subsections, I will look more closely at what could drive the contrarian hypothesis that I propose to model. I want to show that the Democrats having a smaller majority might not be a problem for them. I mentioned that I saw two reasons for that. First, the diffusion of responsibility across defecting Republicans, in a psychological sense, might play a role. I will talk about the related literature in this section. The next section looks to the general idea of a material sanction for deviating.

2.3.1 Diffusion of Responsibility

The seminal paper of Darley and Latane(1968)[16] have shown in a psychological experiment that people will react or take decisions more slowly, or not react at all when they are in a group. The main example in this literature is the assassination of Kitty Genovese in New York in 1964 that was witnessed by 38 people but where, allegedly at least, nobody reacted. I think that such social loafing can occur in the decision of a senator to defect from the party line: Olympia Snowe couldn't really move on her own, but if Scott Brown is now a potential defector, she will feel less compelled to follow the party line because she would feel less responsible for, for instance, breaking the filibuster.

2.3.2 Conformity

One adjacent psychological effect is related to the theory of groups and teams. Several studies, starting with Solomon Asch(e.g, [4]) have shown that when an individual faces disagreement from a group, it will be prone to conform to the group's view even if it looks completely irrational. Those studies have also shown that introducing another dissenter from the group along with the subject has a negative impact on conformity. This effect, thus, could play here: Scott Brown is the extra dissenter from the Republican Party and Olympia Snowe is our subject.

2.3.3 Other theories on groups

In a recent paper, Charness et al.(2007)[12] show that 'salient group membership affects behavior in a strategic environment, even if this membership provides no information and has no effect on payoffs.'

2.4 Punishment

In the literature on game theory, punishment has often been used in repeated games in order to entice cooperation by deterrence. Abbink et al.(2009)[1] find that punishment is more of a backward looking process, in which agents are punished for not respecting social norm rather than in order to deter from future ‘bad’ action. They also show that punishment has a strong negative impact on the heterogeneity between team members. In our framework, this would mean that the ability to punish should affect a Senator’s propensity to defect not only through a material channel. Other papers(e.g. Goette et al.(2006)[24]) have shown that in a prisoner’s dilemma in a group framework, the impact of belonging in a group was featured mostly when a third-party could punish a out-group member who was not cooperating with an in-group member. What this literature tells us is that the relevance of punishment depends on the identity of the punisher and the saliency of the relation between the punisher and the subject.

3 Modelling the problem

3.1 Basic structure

In this section, I present the first way in which we thought of solving the problem. Some further simplifications will be made later on.

Outcome One bill is on the floor. We denote the outcome of the vote as x . If $x = 1$, the bill is passed, if $x = 0$, it is not. I assume that the Democrats, not represented in the game, only need one defection from the G.O.P. in order to pass the bill.

Preferences We now have to define the preferences

- For a Republican, we normalize the utility of filibustering the bill to 1. $c < 1$ simply defines the relative utility if the bill passes. It can encompass things like a lower probability to win midterm election, uneasiness at the policies put in place(e.g. social issues, government spending, ...). The Republican prefers the bill not to pass in any circumstances. Mathematically speaking, we have

$$u_R(x) = \begin{cases} 1 & \text{if } x = 0 \\ c & \text{otherwise} \end{cases}$$

- For the Brownites, the utility depends on the outcome, but also on how they voted/whether they defected to the Democrats or not. Let v denote one’s eventual vote, $v = 1$ if the Brownite voted with the Democrats and $v = 0$ otherwise. We assume that by voting with the Republicans, the senator

alienates his constituency and loses an (exogenous) fraction δ of his/her utility. By voting with the Democrats, he satisfies his constituency but can get punished by the Republicans. Punishment is P , and is distributed among all defectors, whose number is N_D . Another way to see the ‘punishment function’ is if the Republican party was deciding to punish one congressman at random across the defectors. I will talk about the restrictions on the parameters below.

$$u_B(x, v = 0) = \begin{cases} \frac{c}{\delta} & \text{if } x = 0 \\ \frac{1}{\delta} & \text{otherwise} \end{cases} \quad u_B(x, v = 1) = \begin{cases} c - \frac{P}{N_D} & \text{if } x = 0 \\ 1 - \frac{P}{N_D} & \text{otherwise} \end{cases}$$

Parameter restrictions In order to make the experiment interesting, and with some similarities with a public good model, we can set the parameters so that one Senator does not benefit from being the only one to deviate, but that if 2 Senators deviate, they would both be better off. This means that we will have some bounds on P depending on c and δ . Namely, we would have $1 - p \leq \frac{c}{\delta}$ and $1 - \frac{P}{2} \geq \frac{1}{\delta}$

Solving the model I think the model is quite hard to solve quantitatively, and that the equilibria, if any, are non-trivial. I should work on that. However, the experiment should enable us to consider different effects of interest.

To solve the model, we can start by considering that ex-ante symmetry will imply that all players will have the same beliefs about the other player’s type and their propensity to defect.

Assuming that a Brownite puts a probability $p_D(\beta)$ on defecting (which depends on the initial distribution of types, which is common knowledge), one agent’s belief about the number of defectors is a random variable N_D that has a Binomial distribution. If N is the number of initial players, we have

$$N_D \sim B\left(\frac{N-1}{\beta p_D(\beta)}\right)$$

Therefore, the utility of voting for one congressman from defecting, and thus be sure of getting the bill passed, is

$$\begin{aligned} E\{u_B(x, 1)\} &= \sum_{k=0}^{N-1} \mathbb{P}\left(B\left(\frac{N-1}{\beta p_D(\beta)}\right) = k\right) \left(1 - \frac{P}{k+1}\right) \\ &= \sum_{k=0}^{N-1} (\beta p_D(\beta))^k \cdot (1 - \beta p_D(\beta))^{N-1-k} \left(1 - \frac{P}{k+1}\right) \end{aligned}$$

And the utility of voting to support the filibuster is

$$E\{u_B(x, 0)\} = \mathbb{P}\left(B\left(\frac{N-1}{\beta p_D(\beta)}\right) = 0\right) \frac{c}{\delta} + \left[1 - \mathbb{P}\left(B\left(\frac{N-1}{\beta p_D(\beta)}\right) = 0\right)\right] \frac{1}{\delta}$$

The only possibility for members to play a mixed strategy is to have

$$E\{u_B(x, 0)\} = E\{u_B(x, 1)\}$$

This should give us the possible $p_D(\beta)$, parametrized by c and δ .

3.2 Solving for a simple situation: known number of defectors and exogenous punishment

Through this section, I will loosely use the word ‘defection’ to say that a Brownite votes FOR a bill.

We assume that N_D is known, and that p is a parameter.

3.2.1 All Brownites are needed to pass the bill

Assume that we have an even simpler model where in order to pass a bill, all Brownites need to vote for the bill. Assume that there are no feedback from the constituency ($\delta = 1$) and that $c = 0$. The utility of the Brownites are given by:

$$u_B(x, v = 0) = 0 \qquad u_B(x, v = 1) = \begin{cases} -\frac{P}{N_D} & \text{if } x = 0 \\ 1 - \frac{P}{N_D} & \text{otherwise} \end{cases}$$

The left handside just says that the utility would be 0 in any case if the agent does not vote for the bill, since all members are needed.

Nash equilibrium in pure strategies

- If $P > N_D$ then there does not exist any equilibrium where the Brownites actually defect. This is rather uninteresting.
- If $P \leq N_D$, then we have an equilibrium where all Brownites defect (it is clearly a best response to vote for the bill if everybody else does so). In any equilibrium in pure strategy where one of the Brownite does not vote for the bill, it must be that *nobody* actually vote for the bill: this is the only best response.

Nash equilibrium in mixed strategies It is also interesting to consider potential mixed strategies. Let us look at a symmetric equilibrium. Assume that a Brownite plays $v = 1$ with probability $q \in (0, 1)$. Because he/she must be indifferent between the two strategies, it must be that $E(u_B(x, v_B = 1)) = E(u_B(x, v_B = 0))$. The following proposition shows the equation defining the equation defining the equilibria in symmetric strategies:

Proposition 3.1. *The symmetric mixed strategy must satisfy*

$$q^{N_D-1} - \frac{P}{qN_D}(1 - (1 - q)^{N_D}) = 0$$

or

$$f(q) = N_D q^{N_D} - P + P(1 - q)^{N_D} = 0$$

Proof. See Appendix A.1 □

One can show that this equilibrium is unique. It is shown in the proposition

Proposition 3.2.

$$\exists! q^*(N_D) \in (\bar{q}(N_D), 1) | f(q^*(N_D)) = 0$$

$$\text{where } \bar{q}(N_D) = \frac{\left(\frac{P}{N_D}\right)^{\frac{1}{N_D-1}}}{1 + \left(\frac{P}{N_D}\right)^{\frac{1}{N_D-1}}}$$

$q^*(N_D)$ is the unique symmetric equilibrium in mixed strategy.

Proof. See Appendix A.2 □

3.2.2 $K < N_D$ Brownites are needed to pass the bill

We now assume that there are potentially N_D defectors but only $K < N_D$ are needed to pass the bill.

The utility changes from the fact that one can free-ride on the defection of others:

$$u_B(x, v = 0) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{otherwise} \end{cases} \quad u_B(x, v = 1, D) = \begin{cases} -\frac{P}{D} & \text{if } x = 0 \\ 1 - \frac{P}{D} & \text{if } x = 1 \end{cases}$$

where on the right handside, I wrote the utility of voting for the proposal when $D - 1$ Brownites did.

Nash equilibrium in pure strategies

- If $P > N_D$, nobody has any incentive to defect.
- If $P \leq K$, then the best response to the opposing strategy profile s_{-i} for player i is

$$br(s_{-i}) = \begin{cases} \text{defection} & \text{if there are } k-1 \text{ defections} \\ \text{non-defection} & \text{otherwise} \end{cases}$$

What we can see is that the only possible equilibria in pure strategies are

1. Everybody stays put and there is no defection from the party line
 2. There are k defections from the party line
- If $K < P < N_D$, the situation is similar. What we can see is that the only possible equilibria in pure strategies are
 1. Everybody stays put and there is no defection from the party line
 2. There are $\lfloor P \rfloor$ defections from the party line, where $\lfloor P \rfloor$ is the highest integer below P

Nash equilibrium in mixed strategies As before, it is also interesting to consider potential mixed strategies. Let us look at a symmetric equilibrium. Assume that a Brownite plays $v = 1$ with probability $q \in (0, 1)$. Because he/she must be indifferent between the two strategies, it must be that

$$E(u_B(x, v_B = 1)) = E(u_B(x, v_B = 0))$$

Proposition 3.3. *The equilibria in mixed strategy are defined by*

$$g(q) = N_D \binom{N_D - 1}{K - 1} q^K (1 - q)^{N_D - K} - P(1 - (1 - q)^{N_D}) = 0$$

Proof. See Appendix A.3 □

We can prove that an equilibrium does not always exist. In Appendix A.4, I show in details the issues that arise. Basically, there is usually no equilibrium or 2 equilibria. I provide here a sufficient condition for the existence of two equilibria:

Proposition 3.4. *A sufficient condition for the existence of an equilibrium is:*

$$P < \frac{N_D \binom{N_D - 1}{K - 1} \left(\frac{K - 1}{N_D - 1}\right)^K \left(\frac{N_D - K}{N_D - 1}\right)^{N_D - K}}{1 - \left(\frac{N_D - K}{N_D - 1}\right)^{N_D}} = P_{global}$$

Moreover, there usually exists 0 or 2 equilibria in mixed strategies

Proof. See Appendix A.4 □

One graph can be helpful now to understand which equilibrium players are going to play. I show the function $g(q)$ for $N_D = 6$ and different values of K in Figure 2 and ???. I use a punishment of $P = .5$, which yields the existence of mixed strategies for any K .

As an aside, I would like to underline some real life comparisons to the numbers I am using. The $N_D = 6$ assumption can be seen in parallel to the nomination of Justice Sotomayor and Justice Kagan

to the Supreme Court. For Sonia Sotomayor, 9 Republicans voted Yea: Senators Alexander, Bond, Collins, Graham, Gregg, Lugar, Mel Martinez, Snowe, and Voinovich. For Elena Kagan, Alexander, Bond, Voinovich voted against and Martinez was absent (Scott Brown, interestingly, voted against too), for a total of 5 voters.

The two figures represent the same thing, except that in Figure 2 I also plotted the case where everybody is required to defect.

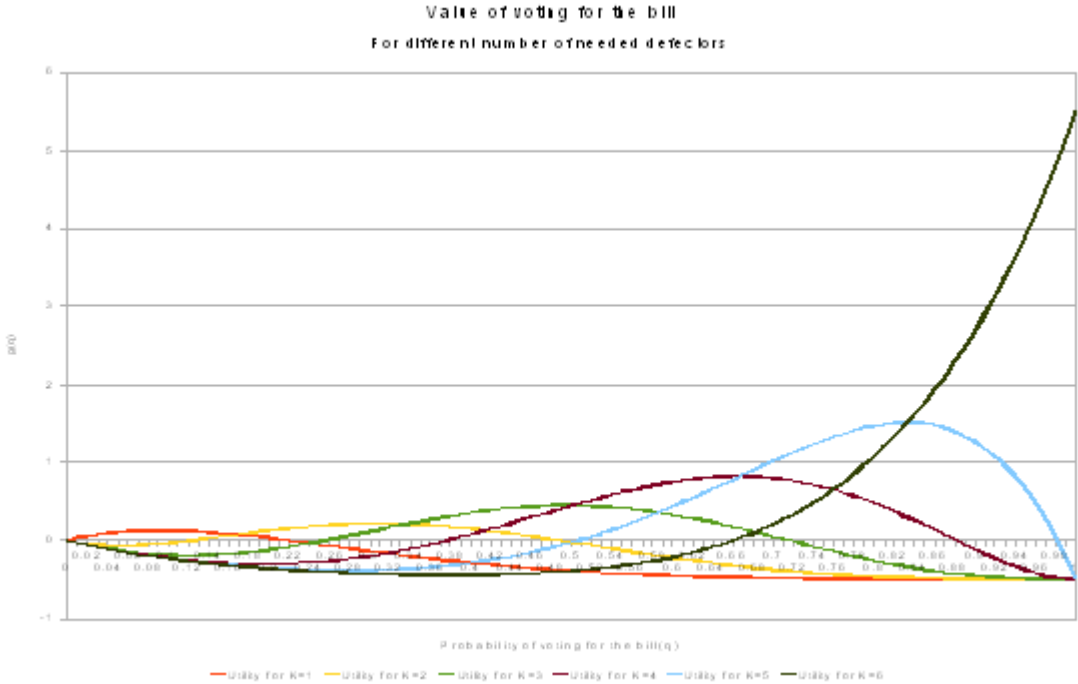


Figure 2: Net utility of voting for the bill for different numbers of required defectors

What we learn from those plots is that there is no clear prediction on whether the increase in the number of defectors required affects the strategies. For the individuals, two effects are competing:

- By increasing the number of needed defectors, it is more difficult to synchronize: for any q , the probability that the required number of defectors is attained is lower.
- By increasing the number of needed defectors, the utility would be higher when the bill is passed because the expected number of defectors is higher, and thus the punishment is divided among more people.

Using a Newton-Raphson algorithm to find the zeros of $g(\cdot)$, we end up with the results in Table 2. I denote q_{inf} and q_{sup} the first and second zeros of the $g(\cdot)$ function.

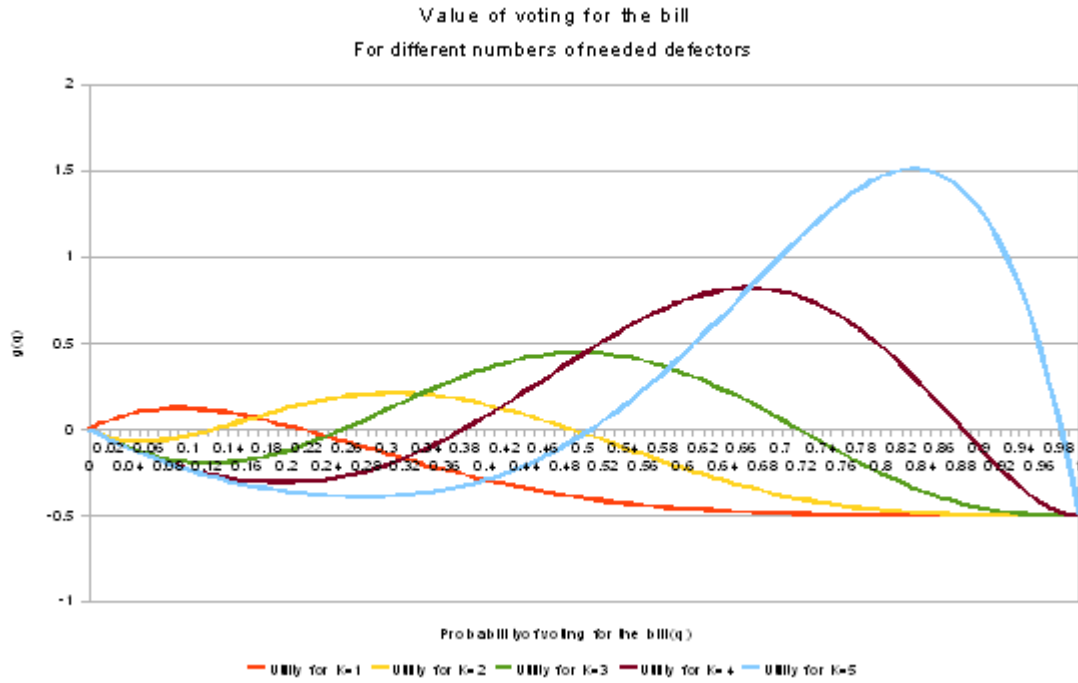


Figure 3: Net utility of voting for the bill for different numbers of required defectors

K	q_{inf}	q_{sup}
1	0.215	0.215
2	0.126	0.488
3	0.256	0.717
4	0.377	0.883
5	0.506	0.982
6	0.661	0.661

Table 1: Possible equilibria in mixed strategies (N=6)

The graphs yield some intuitions. For instance, we can see that as K increases, the slope at the second equilibrium becomes steeper. It is possible that risk-aversion would make it more likely that players play the first equilibrium (with lower probability of passing the bill). However, it seems to me that those higher equilibria are the more stable. Indeed, in the low equilibria, an increase in the probability of voting for the bill yields an incentive for everyone to vote for the bill (because $g'(q) > 0$). In the high equilibria, a similar increase leads to an incentive for the others to not vote for the bill.

It is also interesting that when only one vote is required among 6 possible defectors, the potential defectors are still voting for the bill in 20% of the cases.

We can increase the level of punishment and see what happens. I use the same values as before except that I set $P = 1.1$. The result are in Figure 4. The figures shows that if the number of defectors needed is not high enough, then it will never be optimal to vote for the bill because the threat of punishment is too high, and is not compensated by a likely big spread of the punishment across potential defectors. We can also note that the interval between the two possible equilibria (when they exist) narrows.

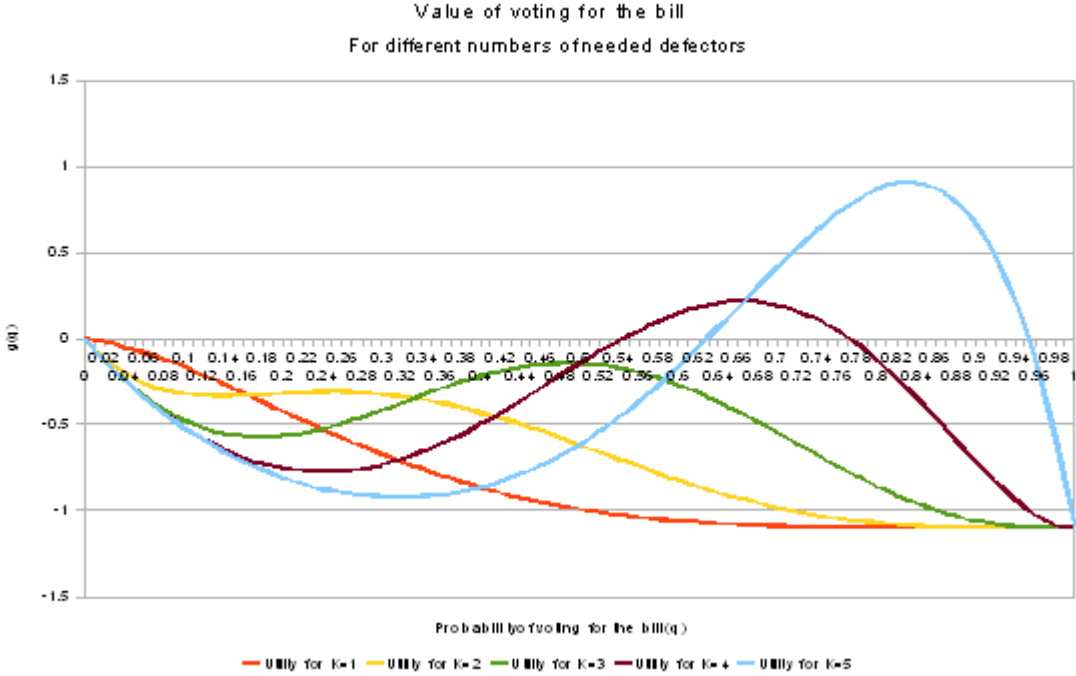


Figure 4: Net utility of voting for the bill for different numbers of required defectors, high punishment

3.2.3 Probability that the bill is passed

With the pure strategy equilibria, the expected number of defectors is trivial. In a symmetric mixed equilibrium $q^*(N_D, K, P)$, the expected number of defectors is

$$E(D) = N_D \cdot q^*$$

The probability that a bill is passed is $b(N_D, K, P)$ such that

$$\begin{aligned} b(N_D, K, P) &= P \left[B \left(q^*(N_D, K, P), N_D \right) \geq K \right] \\ &= \sum_{i=K}^{N_D} \binom{N_D}{i} q^*(N_D, K, P)^i (1 - q^*(N_D, K, P))^{N-i} \end{aligned}$$

In the similar fashion as above, I compute the results for $N = 6$ and different values of K

K	q_{inf}	q_{sup}
1	76.6%%	76.6%
2	16.9%	87.9%
3	17.9%	94.2%
4	14.9%	97.6%
5	11.5%	99.5%
6	8.3%	8.3%

Table 2: Probability that the good is allocated

One interesting thing is that there is no clear relation between the number of required defectors and the probability that the bill is passed. Indeed, if we assume that the low equilibrium is played, we can see that the maximum probability (out of the case $K = 1$ where any member is pivotal) is when $K = 3$. Hence, an increase in the number of required votes can increase the probability of the bill to be passed. This means that the election of Scott Brown might be actually beneficial for the Democrats, which is the main issue of this proposal!

4 Using private value of passing the bill

In order to have pure strategies, which are easier to test in experiments, we are doing a slight change to the model by introducing a private value of passing the bill. Namely, agents are drawing a value $\delta_i \sim F(\delta)$ of passing the bill, and the utilities become

$$u_{i,B}(x, v = 0 | \delta_i) = \begin{cases} 0 & \text{if } x = 0 \\ \delta_i & \text{otherwise} \end{cases} \quad u_{i,B}(x, v = 1, D | \delta_i) = \begin{cases} -\frac{P}{D} & \text{if } x = 0 \\ \delta_i - \frac{P}{D} & \text{if } x = 1 \end{cases}$$

where, for clarification, I index the private value by i to indicate that it may differ across agents. Assume that people are playing a symmetric cutoff strategy, such that

$$\begin{aligned} v = 1 &\Leftrightarrow \delta_i > \hat{\delta} \\ v = 0 &\Leftrightarrow \delta_i < \hat{\delta} \end{aligned}$$

Then, we can do similar calculations as before. I report the computations in Appendix ?? We can first find the equation that defines the equilibrium.

Proposition 4.1. *The symmetric cutoff is given by*

$$\boxed{\gamma(\hat{\delta}) = 0}$$

where

$$\gamma(\delta_i) = \delta_i \binom{N_D - 1}{K - 1} (1 - F(\hat{\delta}))^{K-1} F(\hat{\delta})^{N_D - K} - \frac{P}{(1 - F(\hat{\delta}))N_D} (1 - F(\hat{\delta})^{N_D})$$

Proof. See Appendix B.1 □

Now, we have the same issues to prove the existence of equilibria. The appendix goes through the calculations.

Proposition 4.2. *A sufficient condition for the existence of an equilibrium is*

$$\boxed{b(x_1) \geq 0}$$

where

$$b(x) = Nx \binom{N_D - 1}{K - 1} (1 - F(x))^K F(x)^{N_D - K} - P(1 - F(x)^{N_D}) = 0$$

and

x_1 is defined as the lowest of the two values where $b'(\cdot)$ is zero.

$$x_1 \leq x_2 | b'(x_1) = b'(x_2) = 0$$

Proof. See Appendix B.2 □

As before, there will usually be either 2 or 0 equilibrium in cutoff strategies.

To see what is happening for a simple case, I use some examples similar to what I used before.

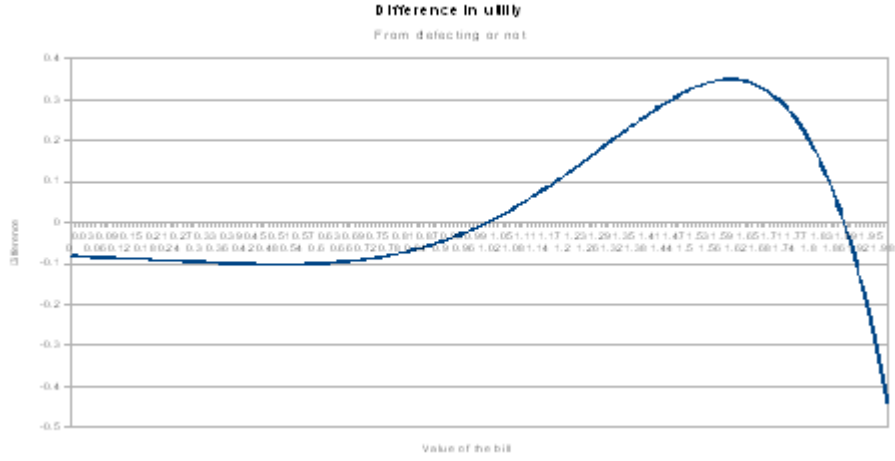


Figure 5: Net utility of voting for the bill, $K = 2$, $M = 2$

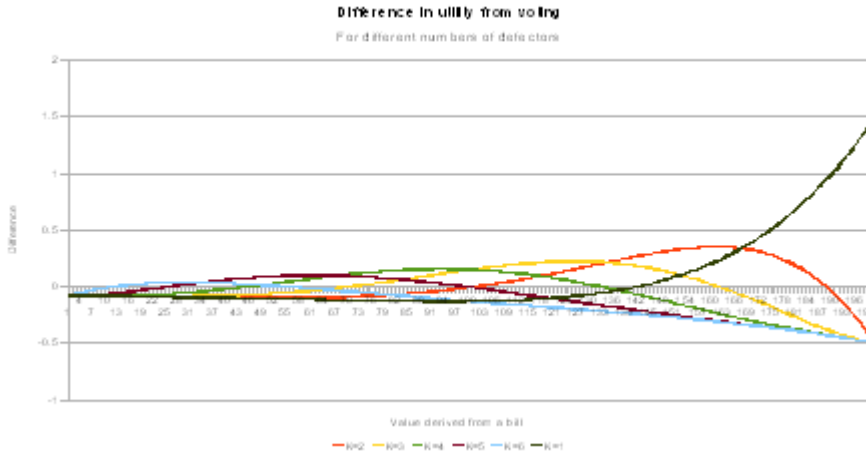


Figure 6: Net utility of voting for the bill, multiple K , low punishment

K	δ_{inf}	δ_{sup}
1	1.41	1.41
2	1.01	1.885
3	0.7	1.62
4	0.45	1.32
5	0.25	0.985
6	0.12	0.535

Table 3: Possible cutoffs($N=6$), low punishment

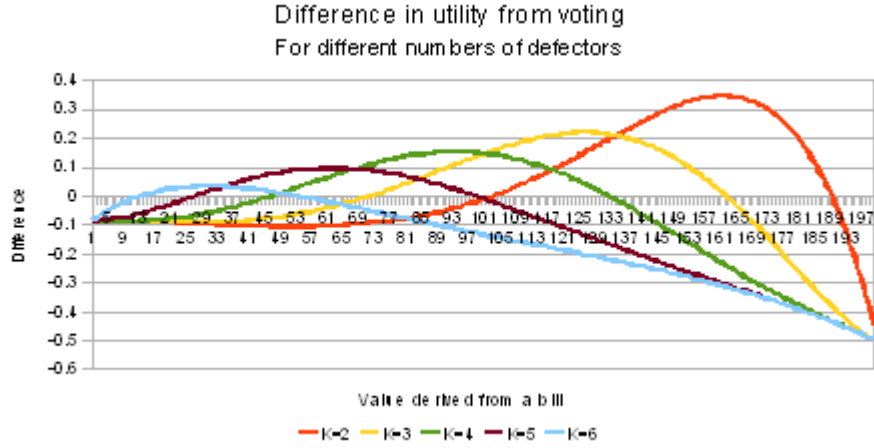


Figure 7: Net utility of voting for the bill, multiple K , low punishment

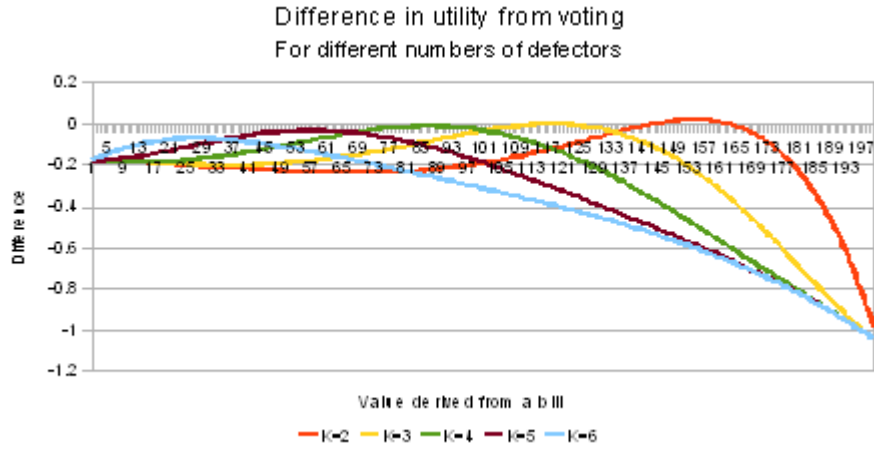


Figure 8: Net utility of voting for the bill, multiple K , high punishment

K	δ_{inf}	δ_{sup}
1	1.47	1.47
2	1.085	1.855
3	0.76	1.565
4	0.5	1.26
5	0.3	0.91
6	0.175	0.43

Table 4: Possible cutoffs($N=6$), low punishment a little bit higher

K	δ_{inf}	δ_{sup}
1	1.685	1.685
2	1.43	1.63
3	None	None
4	None	None
5	None	None
6	None	None

Table 5: Possible cutoffs(N=6), high punishment

I report the cutoff points, for $N = 6$ and low punishment($P = .5$) in Table 4 and with high punishment($P = 1.05$) in Table 5

We can see that with low punishment, there will always be some cutoff equilibria. However, a high punishment makes the outside option really costly. Nobody can free-ride because the cutoffs are quite high, so that there is a strong possibility that the bill does not pass. This means that one should vote for the bill. But the punishment being too high, coupled with the high cutoff(i.e. low probability of the bill passing), makes this option risky.

4.1 Probability that the bill is passed

With the pure strategy equilibria, the expected number of defectors is trivial. In a symmetric bayesian equilibrium with cutoff strategy $\hat{\delta}(N_D, K, P, M) = \hat{\delta}$, the expected number of defectors is:

$$E(D) = N_D(1 - F(\hat{\delta}))$$

The ex-ante probability that one individual defects is $1 - F(\hat{\delta})$ Hence, The probability that a bill is passed is $b(N_D, K, P)$ such that

$$\begin{aligned} b(N_D, K, P) &= P \left[B \left(\begin{matrix} N_D \\ 1 - F(\hat{\delta}) \end{matrix} \right) \geq K \right] \\ &= \sum_{i=K}^{N_D} \binom{N_D}{i} (1 - F(\hat{\delta}))^i F(\hat{\delta})^{N-i} \end{aligned}$$

In the similar fashion as above, I compute the results for $N = 6$ and different values of K . With weak punishment

K	δ_{inf}	δ_{sup}
1	87.8%	87.8%
2	88.6%	4.2%
3	88.3%	8.7%
4	86.8%	10.7%
5	83.3%	11.7%
6	69%	15.4%

Table 6: Probability that the bill passes

K	δ_{inf}	δ_{sup}
1	84.2%	84.2%
2	84.5%	6.5%
3	84.7%	12.2%
4	83.1%	14%
5	77.6%	15.7%
6	57.7%	23.4%

Table 7: Probability that the bill passes with a little bit higher punishment

K	δ_{inf}	δ_{sup}
1	60.9%	60.9%
2	54.7%	30.8%
3	None	None
4	None	None
5	None	None
6	None	None

Table 8: Probability that the bill passes with high punishment

4.2 Comparative statics in the uniform case

4.2.1 Change of cutoffs with punishment

We can directly do comparative statics on the punishment level.

Proposition 4.3. *The low cutoff should increase with punishment, and the high cutoff should decrease with punishment.*

Proof. See Appendix B.2 □

The intuition is that the low cutoff is too low: the bill does not provide enough benefit in expectation to match the punishment. The high cutoff: now, if everybody plays high cutoff, one wants to use an even higher cutoff because the downside is becoming risky. The only way one can sustain an equilibrium is actually to lower the cutoff in order to have less punishment in expectation.

4.3 Distribution assumption

To look at the sensitivity of the cutoff equilibria to the distribution, we can derive some conditions when the distribution is given by a general form like $F(x) = x^b, b \in (0, +\infty)$.

Remember that

$$\gamma(x) = 0 \Leftrightarrow b(x) = Nx \binom{N_D - 1}{K - 1} (1 - F(x))^K F(x)^{N_D - K} - p(1 - F(x)^{N_D}) = 0$$

so that

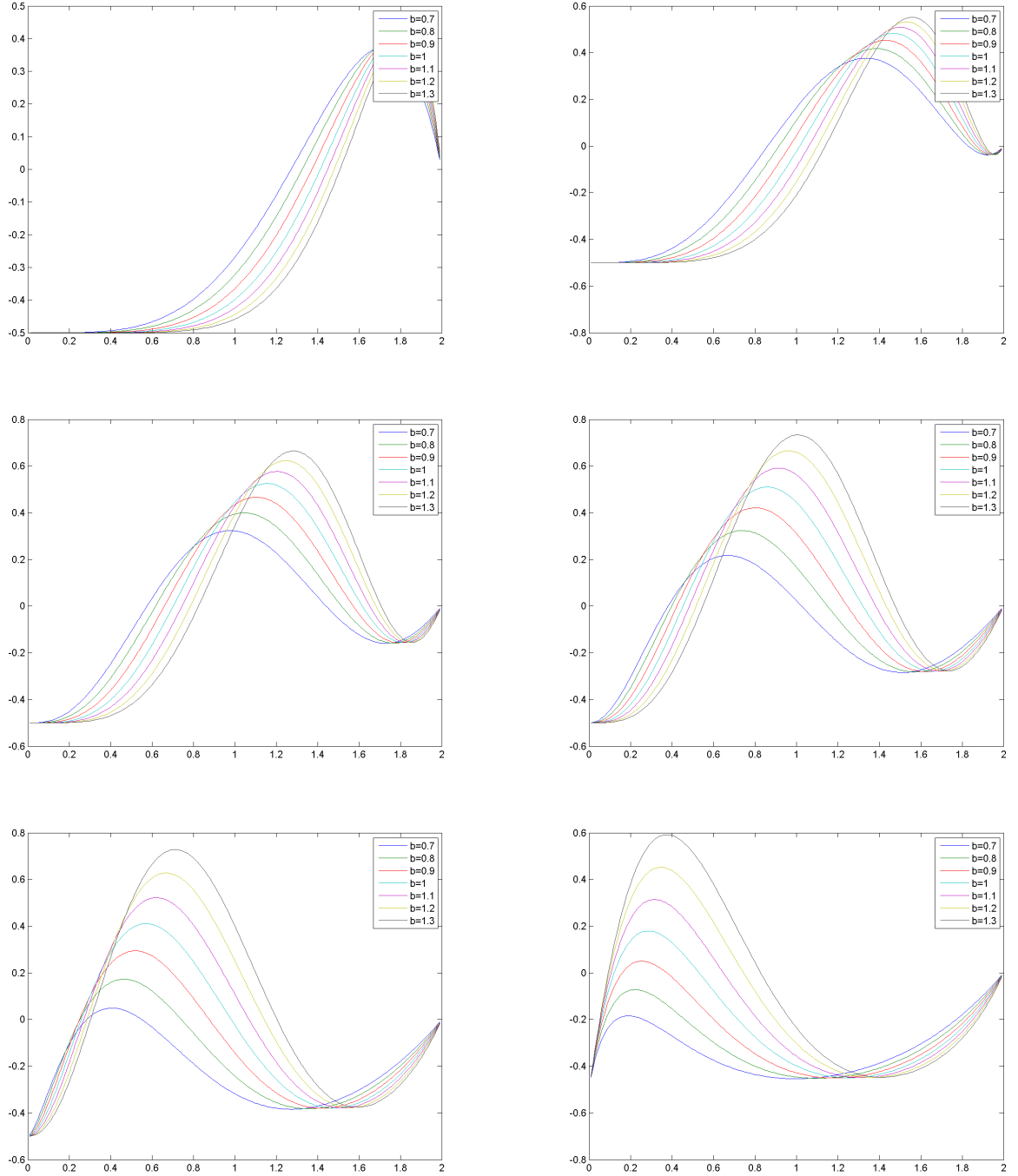
$$\begin{aligned} b'(x) = 0 \Leftrightarrow & \binom{N_D - 1}{K - 1} (1 - x^b)^K x^{b(N - K)} - \binom{N_D - 1}{K - 1} K (1 - x^b)^{K - 1} b x^{b + bN - bK} \\ & + b(N - K) \binom{N_D - 1}{K - 1} (1 - x^b)^K x^{bN - bK} + P b x^{N b - 1} = 0 \end{aligned}$$

Figure 9 show the change in the γ function depending on the distribution assumptions, in the case $N = 6$. There are 7 curves for $b \in [.7, 1.3]$, in .1 increments. What we see is that the cutoffs are increasing with b .

5 Correlated Values

In this section, I introduce correlated values for the bill. This could be justified both theoretically, practically and empirically. On the theory and practicability, we saw that one issue with the results above are that we end up with multiple equilibria. Moreover, it is quite difficult to pin down the stability properties of those equilibria. Introducing correlation, through the use of ideas from Harsanyi to the

Figure 9: Changing distribution, difference in utility, $N = 6$, for different b



global games literature, has been shown to reduce the number of equilibria. From an empirical point of view, it seems justifiable to consider that congressmen derive similar values from a bill, meaning that if a bill is very good for one, it is likely to be very good for another. For an extreme example, it seems that if a bill is good for Olympia Snowe, it must be quite good for Susan Collins.

5.1 Same value for the bill, observed with noise

The utilities take the same form as above. However, I come back to a common value setting, where the common value is given by θ . The only difference will be that congressmen will observe their value with noise.

$$u_{i,B}(x, v = 0|\theta) = \begin{cases} 0 & \text{if } x = 0 \\ \theta & \text{otherwise} \end{cases} \quad u_{i,B}(x, v = 1, D|\delta_i) = \begin{cases} -\frac{P}{D} & \text{if } x = 0 \\ \theta - \frac{P}{D} & \text{if } x = 1 \end{cases}$$

Indeed, I assume that agent i observes a valuation δ_i , where $\delta_i = \epsilon_i^\sigma \theta$. ϵ_i is the noise, and I assume that $\ln(\epsilon_i) \sim N(0, 1)$. Hence σ will measure the amount of uncertainty. I finally assume that the prior on θ is completely uninformative and degenerate: $\theta \sim U(\mathbb{R}^+)$

I used this structure because I wanted to work with positive valuations. The multiplicative framework, in logs, yields summation and the Bayesian updating becomes easier, as you will see below. However, I found out that using something of the form $\delta_i = \theta + \epsilon_i$ and working with normal distributions actually yields similar results.

After having observed δ_i , the posterior for θ is $\theta|\delta_i \sim \text{lognorm}(\ln(\delta_i), \sigma^2)$. Likewise, and that is where the correlation is playing, agent i has some beliefs on the others' valuations. Specifically, by Bayesian updating, the belief about the valuation observed by agent j is given by $\delta_j \sim \text{lognorm}(\ln(\delta_i), 2\sigma^2)$.

Let us look at a symmetric equilibrium cutpoint strategy. Assume that congressmen vote if, and only if, they observe $\delta > \delta^*$. When it exists, the equilibrium in cutpoint strategy is now unique.

Proposition 5.1. *When it exists, the equilibrium in cutpoint strategy is unique and given by*

$$\delta^* = \frac{p}{n \binom{n-1}{k-1}} (2^n - 1) e^{-\sigma^2}$$

Proof. See Appendix C.1 □

However, the fact that, when the other voters are playing a cutpoint strategy, I should play one, is not completely trivial. With independent values, the probability that a certain number of congressmen vote is independent of one's own value. Hence, it is trivial that if my bill's value is higher, I will get higher utility. Here, there is a second effect. When I see a high value, I think that people will see high values too. If they play a cutpoint strategy, the probability that each of them vote is higher. The probability

to have the exact number of votes that makes me pivotal depends on what I observe. Now, if I observe a high value, the probability of a big turnout is high, and the probability of a low turnout is low. Hence, if K is high, I will definitely gain. If K is low, the utility gain from voting is not substantial.

Indeed, one can show that in this setting, **playing a cutpoint strategy is not a best response to other players playing a cutpoint strategy**

In order to get rid of the free-riding effect, I introduce another variable. Specifically, I argue that k defectors are required to pass the bill, but when exactly k Republicans defect, the probability of the bill passing is not 1. Now, the probability of the bill passing is increasing in the number of voters. In practice, we can think of some potential defections from the Democratic parties that would have to be compensated.

This helps counteract the free-riding effect. This is the case because now, a voter is pivotal in more cases: anytime the number of other voters is at least $k - 1$, the individual can change the expected outcome. One can then show that

Proposition 5.2. *In the setting described above, where the probability of the bill passing when $k' \geq k$ Republicans defect is $\frac{k'}{n}$, a cutpoint strategy is a best response to the other players playing a cutpoint strategy. Moreover*

The equilibrium in cutpoint strategy is unique and given by

$$\delta^* = \frac{p}{\sum_{i=k}^{n-1} \binom{n-1}{i}} (2^n - 1) e^{-\sigma^2}$$

5.2 Probability of the bill passing

We can now look at the probability of the bill passing. Given the assumption of complete ignorance on the true value of the bill, the ex-ante probability of the bill passing is not well defined. Therefore, I will look at the ‘ex-interim’ probability. The first question is: given my observation, what is my expectation of the bill passing? The second related question, which is our main interest is: how does this probability varies with n ?. First, let us define the ex-interim probability of voter j voting when I observe δ as $\eta(\delta)$.

$$\begin{aligned} \eta(\delta) &= P(\delta_j > \delta^* | \delta) \\ &= 1 - \Phi\left(\frac{\ln(\delta^*) - \ln(\delta)}{\sqrt{2\sigma^2}}\right) \end{aligned}$$

where $\Phi(\cdot)$ is the standard normal cdf.

Define p_i^n as the probability that the bill passes when there are n congressmen, and i defectors

The probability of a bill passing given δ is thus defined by $\Gamma(\delta)$ such that:

$$\Gamma(\delta) = \begin{cases} \sum_{j=k}^{N-1} p_j^n \binom{n-1}{j} \eta(\delta)^j (1 - \eta(\delta))^{n-1-j} & \text{if } \delta < \delta^* \\ \sum_{j=k-1}^{N-1} p_{j+1}^n \binom{n-1}{j} \eta(\delta)^j (1 - \eta(\delta))^{n-1-j} & \text{if } \delta > \delta^* \end{cases}$$

In Figure 10, I show the probability of the bill passing for $n = 8$, and different values of k . We can see that the probability of the bill passing is indeed falling as the number of required defectors increases. However, the switch is not that strong for $k = 2$ to $k = 3$ for instance

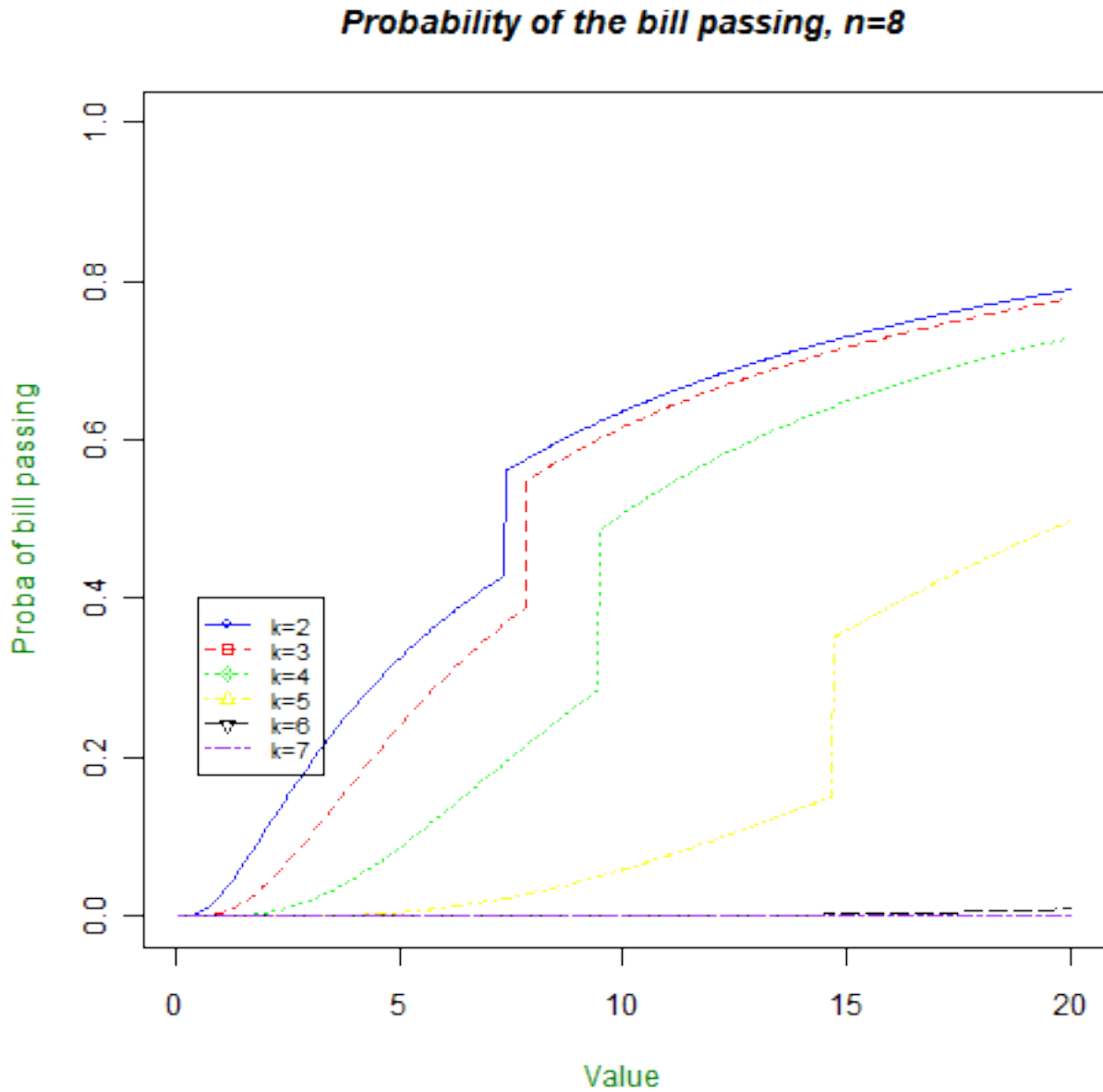


Figure 10: Probability of the bill passing, 8 voters, k required defectors

In Figure 11, I do the same thing, except that I consider that the gap between the number of centrist Republicans and the required defectors is held constant. I hold it at 4 here, and I look at n between 6 and 12.

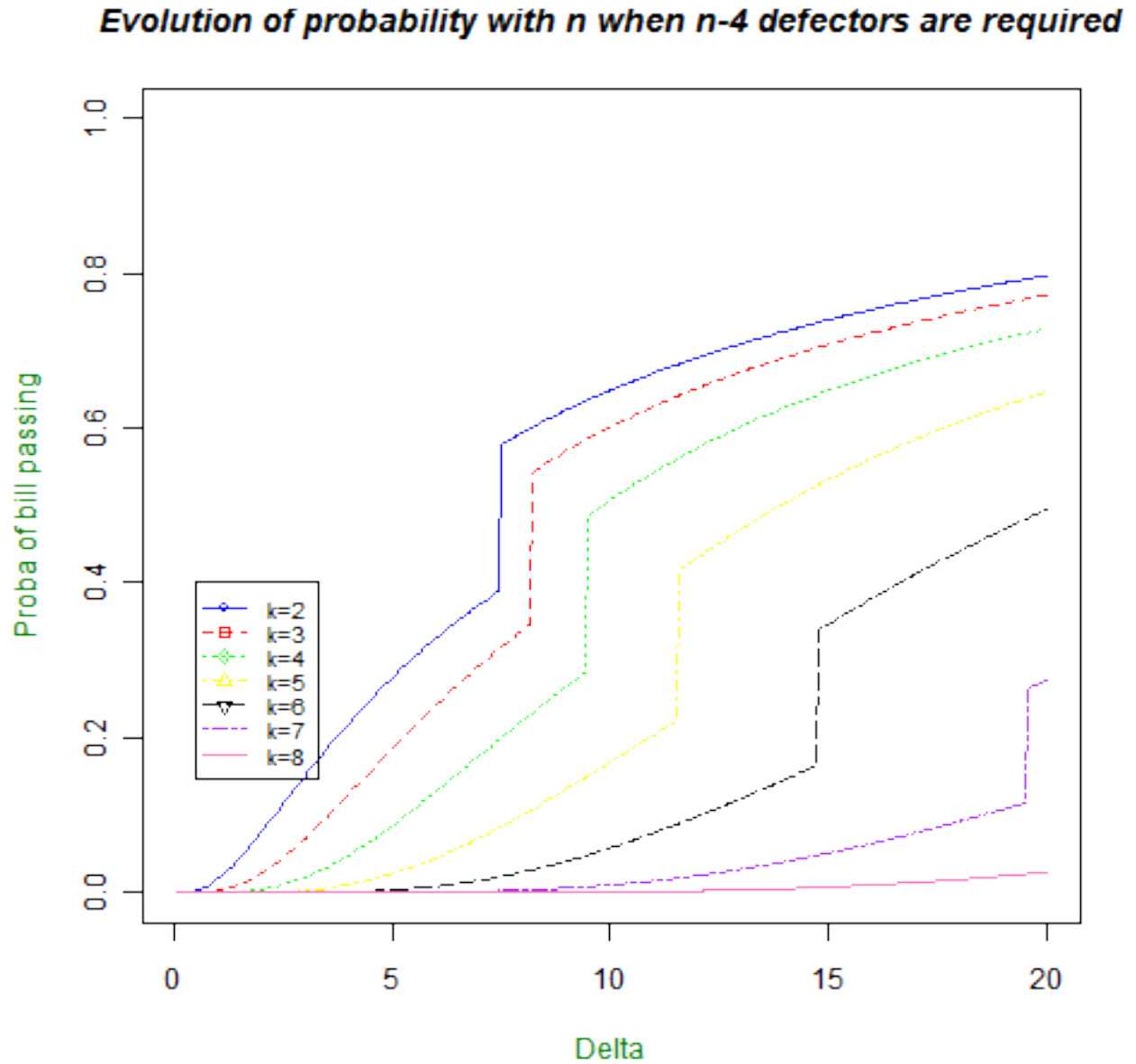


Figure 11: Probability of the bill passing, n voters, $n-4$ required defectors

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A Common value

A.1 Equilibrium definition

Proposition A.1. *The symmetric mixed strategy must satisfy*

$$q^{N_D-1} - \frac{P}{qN_D}(1 - (1-q)^{N_D}) = 0$$

or

$$f(q) = N_D q^{N_D} - P + P(1-q)^{N_D} = 0$$

Proof. First, note that $E(u_B(x, v_B = 0)) = 0$ We can compute $E(u_B(x, v_B = 1))$

$$\begin{aligned} E(u_B(x, v_B = 1)) &= q^{N_D-1} \left(1 - \frac{P}{N_D}\right) + \sum_{k=0}^{N_D-2} \left[\binom{N_D-1}{k} q^k (1-q)^{N_D-1-k} \frac{-P}{k+1} \right] \\ &= q^{N_D-1} - \sum_{k=0}^{N_D-1} \left[\binom{N_D-1}{k} q^k (1-q)^{N_D-1-k} \frac{P}{k+1} \right] \\ &= q^{N_D-1} - \sum_{k=0}^{N_D-1} \left[\frac{(N_D-1)!}{k!(N_D-1-k)!} q^k (1-q)^{N_D-1-k} \frac{P}{k+1} \right] \\ &= q^{N_D-1} - \frac{P}{qN_D} \sum_{k=0}^{N_D-1} \left[\frac{N_D!}{(k+1)!(N_D-1-k)!} q^{k+1} (1-q)^{N_D-1-k} \right] \\ &= q^{N_D-1} - \frac{P}{qN_D} \sum_{k=0}^{N_D-1} \left[\binom{N_D}{k+1} q^{k+1} (1-q)^{N_D-(1+k)} \right] \\ &= q^{N_D-1} - \frac{P}{qN_D} \sum_{k=1}^{N_D} \left[\binom{N_D}{k} q^k (1-q)^{N_D-k} \right] \\ E(u_B(x, v_B = 1)) &= q^{N_D-1} - \frac{P}{qN_D}(1 - (1-q)^{N_D}) \end{aligned}$$

□

A.2 Unicity of the MSNE

Proposition A.2.

$$\boxed{\exists! q^*(N_D) \in (\bar{q}(N_D), 1) | f(q^*(N_D)) = 0}$$

$$\text{where } \bar{q}(N_D) = \frac{\left(\frac{P}{N_D}\right)^{\frac{1}{N_D-1}}}{1 + \left(\frac{P}{N_D}\right)^{\frac{1}{N_D-1}}}$$

$q^*(N_D)$ is the unique symmetric equilibrium in mixed strategy.

Proof.

$$\begin{aligned}
f'(q) = 0 &\leftrightarrow N_D^2 q^{N_D-1} - N_D P (1-q)^{N_D-1} = 0 \\
&\leftrightarrow \frac{q}{1-q} = \left(\frac{P}{N_D}\right)^{\frac{1}{N_D-1}} \\
&\leftrightarrow q = \frac{\left(\frac{P}{N_D}\right)^{\frac{1}{N_D-1}}}{1 + \left(\frac{P}{N_D}\right)^{\frac{1}{N_D-1}}} \\
f'(q) < 0 &\leftrightarrow q < \frac{\left(\frac{P}{N_D}\right)^{\frac{1}{N_D-1}}}{1 + \left(\frac{P}{N_D}\right)^{\frac{1}{N_D-1}}}
\end{aligned}$$

Because $\frac{q}{1-q}$ is an increasing function of q , $\frac{q}{1-q}|_{q=0} = 0$ and $\lim_{q \rightarrow 1} \frac{q}{1-q} = +\infty$, we can see that

$$\forall N_D > P, \exists! \bar{q}(N_D) \in (0, 1) | f'(\bar{q}(N_D)) = 0$$

$$\bar{q}(N_D) = \frac{\left(\frac{P}{N_D}\right)^{\frac{1}{N_D-1}}}{1 + \left(\frac{P}{N_D}\right)^{\frac{1}{N_D-1}}}$$

and

$$\boxed{\exists! q^*(N_D) \in (\bar{q}(N_D), 1) | f(q^*(N_D)) = 0}$$

$q^*(N_D)$ is the unique symmetric equilibrium in mixed strategy. □

A.3

Proposition A.3. *The equilibria in mixed strategy are defined by*

$$g(q) = N_D \binom{N_D-1}{K-1} q^K (1-q)^{N_D-K} - P(1 - (1-q)^{N_D}) = 0$$

Proof. We have that

$$\begin{aligned}
E(u_B(x, v_B = 1)) &= \sum_{k=K-1}^{N_D-1} \left[\binom{N_D-1}{k} q^k (1-q)^{N_D-1-k} \left(1 - \frac{P}{k+1}\right) \right] + \sum_{k=0}^{K-2} \left[\binom{N_D-1}{k} q^k (1-q)^{N_D-1-k} \frac{-P}{k+1} \right] \\
&= \sum_{k=K-1}^{N_D-1} \left[\binom{N_D-1}{k} q^k (1-q)^{N_D-1-k} \right] - \frac{P}{q N_D} \sum_{k=1}^{N_D} \left[\binom{N_D}{k} q^k (1-q)^{N_D-k} \right] \\
E(u_B(x, v_B = 1)) &= \sum_{k=K-1}^{N_D-1} \left[\binom{N_D-1}{k} q^k (1-q)^{N_D-1-k} \right] - \frac{P}{q N_D} (1 - (1-q)^{N_D})
\end{aligned}$$

$$E(u_B(x, v_B = 0)) = \sum_{k=K}^{N_D-1} \left[\binom{N_D-1}{k} q^k (1-q)^{N_D-1-k} \right]$$

Hence, the symmetric mixed strategy equilibrium must satisfy

$$\binom{N_D-1}{K-1} q^{K-1} (1-q)^{N_D-K} - \frac{P}{qN_D} (1 - (1-q)^{N_D}) = 0$$

or

$$g(q) = N_D \binom{N_D-1}{K-1} q^K (1-q)^{N_D-K} - P(1 - (1-q)^{N_D}) = 0$$

Note that $g(0) = 0$ and $g(1) = -P$. □

A.4 Condition for the existence of mixed equilibria

Proposition A.4. *There exists an equilibrium if, for instance:*

$$P < \frac{N_D \binom{N_D-1}{K-1} \left(\frac{K-1}{N_D-1}\right)^K \left(\frac{N_D-K}{N_D-1}\right)^{N_D-K}}{1 - \left(\frac{N_D-K}{N_D-1}\right)^{N_D}} = P_{global}$$

Proof. Note that $g(0) = 0$ and $g(1) = -P$.

Now,

$$\begin{aligned} g'(q) = 0 &\leftrightarrow N_D \binom{N_D-1}{K-1} K q^{K-1} (1-q)^{N_D-K} - (N_D-K) N_D \binom{N_D-1}{K-1} q^K (1-q)^{N_D-K-1} - P N_D (1-q)^{N_D-1} = 0 \\ &\leftrightarrow \binom{N_D-1}{K-1} K \left(\frac{q}{1-q}\right)^{K-1} - (N_D-K) \binom{N_D-1}{K-1} \left(\frac{q}{1-q}\right)^K = P \end{aligned}$$

Denote $x = \frac{q}{1-q}$.

$$g'(q) = 0 \leftrightarrow \binom{N_D-1}{K-1} K x^{K-1} - (N_D-K) \binom{N_D-1}{K-1} x^K = P$$

Denote

$$h(x) = \binom{N_D-1}{K-1} K x^{K-1} - (N_D-K) \binom{N_D-1}{K-1} x^K$$

So that $g'(q) < 0 \Leftrightarrow h(x) < P$

Note that

$$h'(x) = (K-1) \binom{N_D-1}{K-1} K x^{K-2} - K(N_D-K) \binom{N_D-1}{K-1} x^{K-1}$$

Hence,

$$\begin{aligned}
h'(x) = 0 &\leftrightarrow (K-1)x^{K-2} - (N_D - K)x^{K-1} = 0 \\
&\Leftrightarrow (K-1) - (N_D - K)x = 0 \\
&\Leftrightarrow x = \frac{K-1}{N_D - K} \\
h'(x) < 0 &\leftrightarrow x > \frac{K-1}{N_D - K}
\end{aligned}$$

Hence, $h(\cdot)$ has a maximum at $\bar{x} = \frac{K-1}{N_D - K}$.

Note that $h(0) = 0$ (provided $K > 1$) and $h(+\infty) = -\infty$.

Hence, if $h(\bar{x}) < P$, then $g'(q) < 0 \forall q$ and there is no symmetric mixed equilibrium. If $h(\bar{x}) > P$ then

$$\exists x_1 < x_2 \in (0, \infty) | h(x_1) = h(x_2) = P$$

If $K = 1$ then one can work out that $g(\cdot)$ has a unique maximum at $q = \frac{N-PN}{N^2-PN}$. From now on, we will always assume that $K > 1$

Denote $q_i = \frac{x_i}{1+x_i}$. Then, $g'(q_i) = 0 \forall i$ and $g(\cdot)$ has a local maximum at q_2 . Hence, there are three cases.

- If $g(q_2) > 0$ then there are two symmetric equilibria in mixed strategies
- If $g(q_2) < 0$ then there are no symmetric equilibrium in mixed strategies
- If $g(q_2) = 0$ then the unique equilibrium in mixed strategy is q_2

First, let's see the condition for the existence of a local maximum:

$$\begin{aligned}
h(\bar{x}) > P &\leftrightarrow \left(\frac{N_D - 1}{K - 1} \right) \bar{x}^{K-1} (K - (N_D - K)\bar{x}) > P \\
&\Leftrightarrow \left(\frac{N_D - 1}{K - 1} \right) \bar{x}^{K-1} (K - (K - 1)) > P \\
&\Leftrightarrow \left(\frac{N_D - 1}{K - 1} \right) \bar{x}^{K-1} > P \\
&\Leftrightarrow \left(\frac{N_D - 1}{K - 1} \right) \left(\frac{K - 1}{N_D - K} \right)^{K-1} > P
\end{aligned}$$

So that the condition for the existence of an interior local maximum for $g(\cdot)$ is

$$\boxed{P < \left(\frac{N_D - 1}{K - 1} \right) \left(\frac{K - 1}{N_D - K} \right)^{K-1} = P_{local}}$$

One can note that, for $K = N_D$, this will always be satisfied...

One sufficient condition for the existence of an equilibrium is

$$g(\bar{q}) > 0$$

where $\bar{q} = \frac{\bar{x}}{1+\bar{x}} = \frac{K-1}{N-1}$. This implies that the local maximum is positive, so that it is greater than $g(0)$, and it is a global maximum.

This is equivalent to

$$\begin{aligned} g(\bar{q}) &= N_D \binom{N_D-1}{K-1} \bar{q}^K (1-\bar{q})^{N_D-K} - P(1 - (1-\bar{q})^{N_D}) \\ &= N_D \binom{N_D-1}{K-1} \left(\frac{K-1}{N_D-1}\right)^K \left(\frac{N_D-K}{N_D-1}\right)^{N_D-K} - P \left[1 - \left(\frac{N_D-K}{N_D-1}\right)^{N_D}\right] \end{aligned}$$

Hence, there exists an equilibrium if, for instance:

$$P < \frac{N_D \binom{N_D-1}{K-1} \left(\frac{K-1}{N_D-1}\right)^K \left(\frac{N_D-K}{N_D-1}\right)^{N_D-K}}{1 - \left(\frac{N_D-K}{N_D-1}\right)^{N_D}} = P_{global}$$

In Figures 12, 13 and 14 below, I show the different cases for the function $g(q)$. Only the first graph features a possible mixed equilibrium.

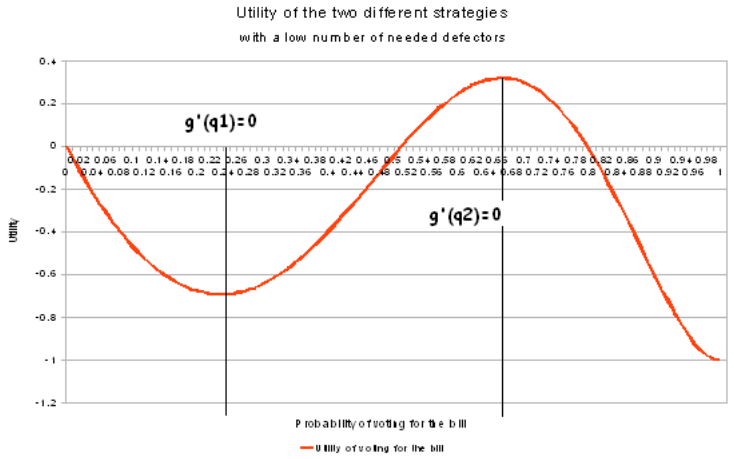
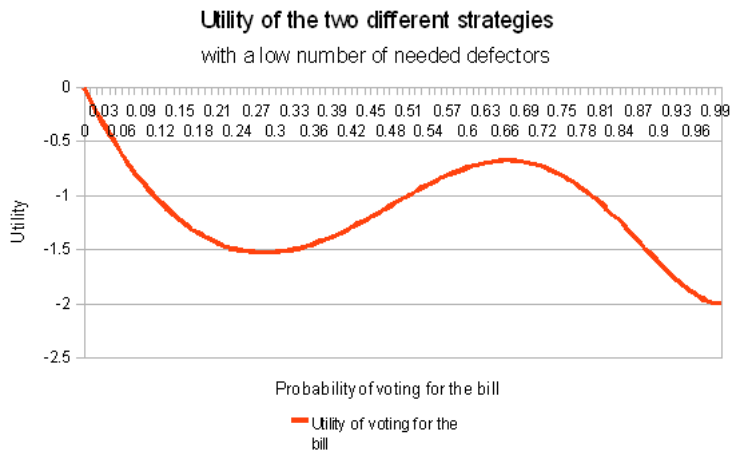


Figure 12: Utility from voting for the bill, $P < \min(P_{global})$

□



3.png

Figure 13: Utility from voting for the bill, $P_{global} \ll P < \min(P_{local})$

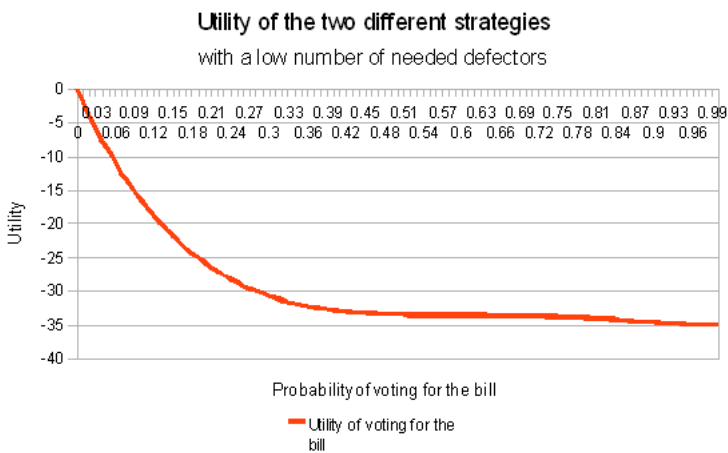


Figure 14: Utility from voting for the bill, $P > P_{local}$

B Independent Heterogeneous Values

B.1 Equilibrium definition

Proposition B.1. *The symmetric cutoff is given by*

$$\gamma(\hat{\delta}) = 0$$

where

$$\gamma(\delta_i) = \delta_i \binom{N_D - 1}{K - 1} (1 - F(\hat{\delta}))^{K-1} F(\hat{\delta})^{N_D - K} - \frac{P}{(1 - F(\hat{\delta}))N_D} (1 - F(\hat{\delta}))^{N_D}$$

Proof. The probability that one votes for the bill ex-post is $1 - F(\hat{\delta})$. Because values are independent, the probability that k agents vote for the bill is independent of one's own value, and is $\binom{N_D - 1}{k} (1 - F(\hat{\delta}))^k F(\hat{\delta})^{N_D - 1 - k}$. We can then compute the value of voting for one Brownite of type δ_i

$$\begin{aligned} E(u_B(x, v_B = 1 | \delta_i)) &= \sum_{k=K-1}^{N_D-1} \left[\binom{N_D - 1}{k} (1 - F(\hat{\delta}))^k F(\hat{\delta})^{N_D - 1 - k} \left(\delta_i - \frac{P}{k+1} \right) \right] + \sum_{k=0}^{K-2} \left[\binom{N_D - 1}{k} (1 - F(\hat{\delta}))^k F(\hat{\delta})^{N_D - 1 - k} \frac{-P}{k+1} \right] \\ &= \delta_i \sum_{k=K-1}^{N_D-1} \left[\binom{N_D - 1}{k} (1 - F(\hat{\delta}))^k F(\hat{\delta})^{N_D - 1 - k} \right] - \frac{P}{(1 - F(\hat{\delta}))N_D} \sum_{k=1}^{N_D} \left[\binom{N_D}{k} (1 - F(\hat{\delta}))^k F(\hat{\delta})^{N_D - k} \right] \\ E(u_B(x, v_B = 0 | \delta_i)) &= \delta_i \sum_{k=K-1}^{N_D-1} \left[\binom{N_D - 1}{k} (1 - F(\hat{\delta}))^k F(\hat{\delta})^{N_D - 1 - k} \right] - \frac{P}{(1 - F(\hat{\delta}))N_D} (1 - F(\hat{\delta}))^{N_D} \end{aligned}$$

Likewise, the value of not voting is given by

$$E(u_B(x, v_B = 0 | \delta_i)) = \delta_i \sum_{k=K}^{N_D-1} \left[\binom{N_D - 1}{k} (1 - F(\hat{\delta}))^k F(\hat{\delta})^{N_D - 1 - k} \right]$$

Therefore, if we look at the difference between the two strategies,

$$\begin{aligned} \gamma(\delta_i) &= E(u_B(x, v_B = 1 | \delta_i)) - E(u_B(x, v_B = 0 | \delta_i)) \\ &= \delta_i \binom{N_D - 1}{K - 1} (1 - F(\hat{\delta}))^{K-1} F(\hat{\delta})^{N_D - K} - \frac{P}{(1 - F(\hat{\delta}))N_D} (1 - F(\hat{\delta}))^{N_D} \end{aligned}$$

which is clearly increasing in δ_i . Hence, agent i plays a cutoff strategy when facing a symmetric cutoff strategy. The symmetric cutoff is given by

$$\gamma(\hat{\delta}) = 0$$

□

B.2 Condition for existence

Proposition B.2. *A sufficient condition for the existence of an equilibrium is*

$$b(x_1) \geq 0$$

where

$$b(x) = Nx \binom{N_D - 1}{K - 1} (1 - F(x))^K F(x)^{N_D - K} - P(1 - F(x)^{N_D}) = 0$$

and

x_1 is defined as the lowest of the two values where $b'(\cdot)$ is zero.

$$x_1 \leq x_2 | b'(x_1) = b'(x_2) = 0$$

Proof.

$$\begin{aligned} E(u_B(x, v_B = 1 | \delta_i)) &= \delta_i P(\text{K-1 or more others defect}) - \sum P(\text{k defectors}) \cdot \frac{P}{k} \\ E(u_B(x, v_B = 0 | \delta_i)) &= \delta_i P(\text{K or more others defect}) \end{aligned}$$

Hence, the difference between the two strategies is increasing in δ_i . Therefore, the optimal strategy *must be* a cutoff strategy.

If we look at a continuous distribution over an interval $[m, M]$, one can see directly that provided $K < N_D$,

$$\gamma(m) = -\frac{P}{N_D} < 0$$

To find $\gamma(M)$, note that

$$\frac{P}{(1 - F(\hat{\delta}))N_D} (1 - F(\hat{\delta})^{N_D}) = \frac{P}{N_D} \sum_{i=0}^{N_D-1} F(\hat{\delta})^i$$

so that provided $K \neq 1$

$$\gamma(M) = -P < 0$$

We want to find if there exists a solution to $\gamma(\hat{\delta}) = 0$. First, note that $\forall x < M$

$$\gamma(x) = 0 \Leftrightarrow b(x) = Nx \binom{N_D - 1}{K - 1} (1 - F(x))^K F(x)^{N_D - K} - P(1 - F(x)^{N_D}) = 0$$

and

$$\gamma(x) < 0 \Leftrightarrow b(x) < 0$$

and we have that $b(m) = -p$, $\lim_{x \rightarrow M} b(x) = 0$

Hence, a sufficient condition for the existence of a cutoff strategy is

$$\sup_{x \in (m, M)} b(x) \geq 0$$

Now,

$$\begin{aligned} b'(x) &= N \binom{N_D - 1}{K - 1} (1 - F(x))^K F(x)^{N_D - K} - NKx \binom{N_D - 1}{K - 1} (1 - F(x))^{K-1} F(x)^{N_D - K} f(x) \\ &+ N(N - K)x \binom{N_D - 1}{K - 1} (1 - F(x))^K F(x)^{N_D - K - 1} f(x) + NPF(x)^{N-1} f(x) \end{aligned}$$

One can see that $\lim_{x \rightarrow M} b'(x) = NpF(M) > 0$, so that we will usually have two or 0 cutoffs.

For $x > m$ and assuming $f(x) > 0$ on $[m, M]$

$$\begin{aligned} b'(x) = 0 &\Leftrightarrow \binom{N_D - 1}{K - 1} (1 - F(x))^K F(x)^{N_D - K} - Kx \binom{N_D - 1}{K - 1} (1 - F(x))^{K-1} F(x)^{N_D - K} f(x) \\ &+ (N - K)x \binom{N_D - 1}{K - 1} (1 - F(x))^K F(x)^{N_D - K - 1} f(x) + PF(x)^{N-1} f(x) = 0 \\ &\Leftrightarrow \binom{N_D - 1}{K - 1} \left(\frac{1 - F(x)}{F(x)} \right)^K \frac{F(x)}{f(x)} - Kx \binom{N_D - 1}{K - 1} \left(\frac{1 - F(x)}{F(x)} \right)^{K-1} \\ &+ (N - K)x \binom{N_D - 1}{K - 1} \left(\frac{1 - F(x)}{F(x)} \right)^K + P = 0 \end{aligned}$$

Assuming a uniform distribution, we have $F(x) = \frac{x-m}{M-m}$ and $f(x) = \frac{1}{M-m}$, so that $\frac{1-F(x)}{F(x)} = \frac{M-x}{x-m}$ and $\frac{F(x)}{f(x)} = x - m$

Then, for x in (m, M) ,

$$\begin{aligned} b'(x) = 0 &\Leftrightarrow \binom{N_D - 1}{K - 1} \left(\frac{M - x}{x - m} \right)^K (x - m) - Kx \binom{N_D - 1}{K - 1} \left(\frac{M - x}{x - m} \right)^{K-1} \\ &+ (N - K)x \binom{N_D - 1}{K - 1} \left(\frac{M - x}{x - m} \right)^K + P = 0 \end{aligned}$$

Putting $m = 0$, and dividing everything by $\frac{(M-x)^{K-1}}{x^{K-1}}$

$$b'(x) = 0 \Leftrightarrow \binom{N_D - 1}{K - 1} (M - x) - Kx \binom{N_D - 1}{K - 1} + (N - K) \binom{N_D - 1}{K - 1} (M - x) + P \left(\frac{x}{M - x} \right)^{K-1} = 0$$

Denote

$$c(x) = \binom{N_D - 1}{K - 1} (M - x) - Kx \binom{N_D - 1}{K - 1} + (N - K) \binom{N_D - 1}{K - 1} (M - x) + P \left(\frac{x}{M - x} \right)^{K-1}$$

Then,

$$\begin{aligned} c'(x) = 0 &\Leftrightarrow \binom{N_D - 1}{K - 1} (-1 - K - (N - K)) + P(K - 1) \left(\frac{x}{M - x} \right)^{K-2} \left(\frac{M - x + x}{(M - x)^2} \right) = 0 \\ &\Leftrightarrow -\binom{N_D - 1}{K - 1} (N + 1) + P(K - 1) \left(\frac{x}{M - x} \right)^{K-2} \left(\frac{M}{(M - x)^2} \right) = 0 \end{aligned}$$

Now, because $\frac{x^{K-2}}{(M-x)^K}$ is an increasing function of x , and goes from 0 to ∞ when x spans $(0, M)$ (check case $K=2$),

$$\exists \bar{x} | c'(\bar{x}) = 0$$

and

$$c'(x) > 0 \Leftrightarrow x > \bar{x}$$

Because $c(0) > 0$ and $c(M) > 0$,

$$\exists \tilde{x} | c(\tilde{x}) = 0 \Leftrightarrow c(\bar{x}) \leq 0$$

If $c(\bar{x}) \leq 0$,

$$\exists x_1 \leq x_2 | b'(x_1) = b'(x_2) = 0$$

and

$$\forall x \in (0, x_1), b'(x) > 0$$

$$\forall x \in (x_2, M), b'(x) > 0$$

$$\forall x \in [x_1, x_2], b'(x) \leq 0$$

Hence, b has a local maximum at x_1 . A sufficient condition to have a symmetric equilibrium in cutoff strategy is

$$b(x_1) \geq 0$$

As before, there will usually be either 2 or 0 equilibrium in cutoff strategies. \square

Comparative statics

Proposition B.3. *The low cutoff should increase with punishment, and the high cutoff should decrease with punishment.*

Proof. Remember that

$$\gamma(\delta_i) = \delta_i \binom{N_D - 1}{K - 1} (1 - F(\hat{\delta}))^{K-1} F(\hat{\delta})^{N_D - K} - \frac{P}{(1 - F(\hat{\delta}))N_D} (1 - F(\hat{\delta}))^{N_D}$$

and the symmetric equilibria satisfy

$$\gamma(\hat{\delta}, p) = 0$$

Assume p increases. Now,

If we derive with respect to P , we actually end up with

$$b'(\hat{\delta}) \frac{\partial \hat{\delta}}{\partial P} = 1 - F(\hat{\delta})^{N_D}$$

Hence, the sign of $\frac{\partial \hat{\delta}}{\partial P}$ is determined by the sign of $b'(\hat{\delta})$. But we saw that in the case of multiple equilibria,

$$b'(\hat{\delta}_{inf}) > 0$$

and

$$b'(\hat{\delta}_{sup}) < 0$$

So that the low cutoff should increase with punishment, and the high cutoff should decrease with punishment. □

C Correlated Values

C.1 Unique cutpoint strategy equilibrium

Proposition C.1. *When it exists, the equilibrium in cutpoint strategy is given by*

$$\delta^* = \frac{p}{n \binom{n-1}{k-1}} (2^n - 1) e^{-\sigma^2}$$

Proof. We can compute the expected utilities from voting and not voting. First of all, let us look at the differential gain. The only way voting impacts the gain is when $K - 1$ other centrists voted. This event occurs with probability

$$p_{k-1}(\delta) = \binom{n-1}{k-1} (1 - G(\delta^*|\delta))^{k-1} G(\delta^*|\delta)^{n-k}$$

where $G(\cdot|\delta)$ is the cdf of $\text{lognorm}(\ln(\delta), 2\sigma^2)$.

Hence, the differential gain from voting is given by

$$\begin{aligned} \Delta g &= \int_0^\infty \theta p_{k-1}(\delta) f(\theta|\delta) d\theta \\ &= p_{k-1}(\delta) E(\theta|\delta) \end{aligned}$$

Given the posterior distribution of θ , $E(\theta|\delta) = \delta e^{\sigma^2}$

Now, voting creates a loss from the punishment. The expected loss is given by

$$\begin{aligned} L &= \sum_0^{n-1} \binom{n-1}{i} (1 - G(\delta^*|\delta))^i G(\delta^*|\delta)^{n-1-i} \frac{p}{i+1} \\ &= \frac{p}{n(1 - G(\delta^*|\delta))} (1 - G(\delta^*|\delta))^n \end{aligned}$$

where I use the same tricks as in the paragraphs with independent values.

Hence, individual i votes if and only if

$$\gamma(\delta) = \Delta(g) - L > 0$$

Now, imagine that the $\gamma(\cdot)$ function above is compatible with a cutpoint strategy. I.e., we need to show that

$$\exists! \delta^* | \gamma(\delta) > 0 \Leftrightarrow \delta > \delta^*$$

If that is the case, then we have a simple formula for δ^* . Note that we will define $\gamma(\delta) = 0 \Leftrightarrow \delta = \delta^*$.

Because $G(\delta^*|\delta^*) = \frac{1}{2}$,

$$\gamma(\delta^*) = \binom{n-1}{k-1} \left(\frac{1}{2}\right)^{n-1} \delta^* e^{\sigma^2} - \frac{2p}{n} \left(1 - \left(\frac{1}{2}\right)^n\right)$$

So that

$$\delta^* = \frac{p}{n \binom{n-1}{k-1}} (2^n - 1) e^{-\sigma^2}$$

□

C.2 Existence of cutpoint strategy equilibrium

Proposition C.2. $\forall K, \exists P > 0$ such that playing a cutpoint strategy is a best response to the other players playing a cutpoint strategy.

Proof. Let us write $\gamma(\cdot)$ completely

$$\gamma(\delta) = \binom{n-1}{k-1} (1 - G(\delta^*|\delta))^{k-1} G(\delta^*|\delta)^{n-k} \delta e^{\sigma^2} - \frac{p}{n(1 - G(\delta^*|\delta))} (1 - G(\delta^*|\delta))^n$$

First, we need to find $\frac{\partial G(\delta^*|\delta)}{\partial \delta}$. One can actually see that

$$\begin{aligned} G(\delta^*|\delta) &= P(\delta_j < \delta^*|\delta) \\ &= P(\ln(\delta_j) < \ln(\delta^*)|\delta) \\ &= \Phi\left(\frac{\ln(\delta^*) - \ln(\delta)}{\sqrt{2\sigma^2}}\right) \end{aligned}$$

where $\Phi(\cdot)$ is the standard normal cdf

So that

$$\frac{\partial G(\delta^*|\delta)}{\partial \delta} = -\frac{1}{\sqrt{2\sigma^2}\delta} \phi\left(\frac{\ln(\delta^*) - \ln(\delta)}{\sqrt{2\sigma^2}}\right)$$

where $\phi(\cdot)$ is the standard normal pdf

Now, let's derive some stuff. I will write $G = G(\delta^*|\delta)$, $g = \frac{\partial G(\delta^*|\delta)}{\partial \delta}$. I also use that

$$\frac{1 - G(\delta^*|\delta)^n}{1 - G(\delta^*|\delta)} = \sum_{i=0}^{n-1} G^i$$

$$\gamma'(\delta) = \binom{n-1}{k-1} e^{\sigma^2} G^{n-k-1} (1 - G)^{k-2} [G(1 - G) + (n - k)(1 - G)g\delta - (k - 1)Gg\delta] - \frac{p}{n} \sum_{i=1}^{n-1} igG^{i-1}$$

One can use the fact that $\Phi(x) = o_{x \rightarrow \infty}(\phi(x))$ to show that $\exists \bar{\delta} \forall \delta > \bar{\delta}, \gamma'(\delta) > 0$. Moreover, it is clear that $\gamma'(\delta) > 0$ for small δ . We can play on p so that $\gamma' > 0$. \square