

Elections, Information, and State-Dependent Candidate Quality*

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Abstract

We study a model of electoral competition where politicians are better informed than the electorate about conditions relevant for policy choice. There are two states of the world. The distribution of voters' preferred policies shifts with the state. The two candidates are both completely office-motivated but differ in state-dependent quality. They observe the true state before simultaneously announcing credible positions. Voters do not observe the state but receive a noisy signal before casting their votes. If the signal is sufficiently informative and unknown to the candidates when they take positions then candidates will, in refined equilibrium, reveal the true state by converging on the median position of this state. Otherwise candidates will not reveal their information and voters will rely solely on their signal when casting their votes. We also show that when the signal of the voters is sufficiently informative then they are all better off if candidates do not observe their signal before taking positions. If the signal is not sufficiently informative then all voters prefer that candidates observe their signal before taking positions. Finally we show that it is possible for all voters to become worse off when the quality of both candidates is increased.

Keywords: Electoral Competition, Uncertainty, Information, Candidate Quality.

JEL Classification: D72, D82.

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1 Introduction

It is a reasonable assumption that politicians are generally better informed than the electorate about conditions relevant for policy choice. First of all they have much stronger incentives than voters to be well informed because their careers depend on how they do as policy makers. Furthermore, they usually have staff to help them receive and process information and sometimes have access to information that is not public, for example information related to national security. Therefore it is highly relevant to study the consequences of this informational asymmetry for the functioning of representative democracy. For example, will politicians' policy positions reveal their information to the electorate such that voters can make a better informed choice in the voting booth? And, if so, will the revealing policy positions be optimal for the voters?

There exists a substantial theoretical literature on electoral competition when candidates have more policy-relevant information than voters. This literature includes Harrington (1993), Roemer (1994), Schultz (1996), Cukierman and Tommasi (1998), and Martinelli (2001). It is generally assumed that politicians are at least partially policy-motivated and this is the main reason why they might not reveal their information to the voters. In this paper we assume that political candidates are fully office-motivated but differ in state-dependent quality. Thus the possible incentive for a candidate to take policy positions that will not reveal information to voters does not arise from policy preferences. Rather it arises from the difference in state-dependent quality. If a candidate is of lower quality than his opponent in the current state then he has an incentive not to inform the voters about the state.

The assumption that candidates differ in state-dependent quality seems relevant in many real elections. For example, one candidate may be better qualified to handle national security and foreign relations while his opponent may be better qualified to handle domestic problems. In that case the first candidate will have a quality disadvantage if domestic problems become seen as relatively more important than national security and foreign policy. Therefore he may take policy positions that do not reveal to voters that domestic problems are most important in the current situation, for example by announcing high spending on national security if he is elected. And then, even if his opponent take positions that are consistent with the true current situation voters are not able to infer the truth because they cannot tell who is "lying". Of course, while candidates may be better informed than the electorate, voters are likely to have some information. So announcing a "wrong policy" also has a downside. Even though the true situation it is not revealed by the candidates' positions voters are more likely than not to come to believe in this situation. And if this happens then the electoral prospects of the disadvantaged candidate are really poor - not only is he the low quality can-

didate, his policy positions are also suboptimal. Our model allows us to explore these considerations rigorously in order to get predictions on when we can expect candidates' policy positions to reveal their information.

We consider a model with one issue, for example how much of a fixed budget to spend on national security. There is an odd number of voters and two candidates who simultaneously announce credible positions. The candidates are purely office-motivated, i.e., their only objective is to maximize the probability of winning. There are two states of the world and the distribution of voters' preferred policies shifts with the state. If the issue is national security spending then the two states could correspond to different levels of threat to national security. In that case it is reasonable to assume that each voter prefers higher spending when the threat is high. The candidates observe the true state before taking positions. Voters do not observe the state but they receive a noisy signal (all voters receive the same signal). We consider both the case where the signal is unknown to the candidates when they take positions and the case where it is known to them. The first case is meant to represent a situation where candidates take positions relatively early in the campaign where there is considerable uncertainty about what information voters will have on election day.

Voters do not only care about policy, they also care about candidate quality. One candidate has a quality advantage in one state and the other candidate has a quality advantage in the other state, for example because they differ in experience and skills. Furthermore there is a stochastic element in voters' evaluation of candidates. Suppose, for example, that the two candidates have announced the same position and that the voters know the true state. Then the high quality candidate wins with a probability that is greater than one half but smaller than one. It is the difference in state-dependent quality between the candidates that creates the central strategic aspect of the model: The candidate with a quality advantage in the true state has an incentive to try to reveal it while the other candidate has an incentive to try not to reveal it.

We solve the model for Perfect Bayesian Equilibria satisfying some additional conditions. We assume that all voters share the same belief function. For equilibria where candidates' positions reveal the state to the voters (*revealing equilibria*) we use a refinement criterion that is essentially a weak, local version of the concept of unprejudiced beliefs (Bagwell and Ramey 1991) which was developed precisely for a signalling game with two senders. For *non-revealing equilibria* we introduce a monotonicity condition on voters' beliefs (after showing that the Intuitive Criterion (Cho and Kreps 1987) does not have any bite in this model).

When candidates do not observe the signal of the voters before taking positions (*Case 1*), candidates reveal the true state if the signal is sufficiently informative. They do so by converging on the median of the true state. This is, of course, also the outcome we would get if the state was observable to voters. If the signal of

the voters is not sufficiently informative then candidates will be polarized. More precisely, each candidate will, irrespective of the state, announce the median position of the state in which he has a quality advantage. Thus the voters cannot infer the true state. The intuition behind these results is as follows. Each candidate can either "tell the truth" by announcing the median of the true state or "lie" by announcing the median of the other state (here we disregard all other positions, in our analysis we of course consider all positions). The candidate who has a quality advantage in the true state has no incentive to lie, so he will always tell the truth. The disadvantaged candidate clearly has an incentive to lie. On the other hand, voters have some information of their own. Thus, even though they cannot infer what the candidates' information is if the disadvantaged candidate lies, they are most likely to believe the advantaged candidate. And then he will have both a quality and a policy advantage. So clearly there is a trade-off for the disadvantaged candidate. When the voters' signal is more informative then it is relatively more expensive (in terms of probability of winning) for him to lie. This is why we get revelation when voters' information is sufficiently informative and non-revelation when it is not.

In the situation where candidates observe the signal of the voters before taking positions (*Case 2*), the true state of the world is never revealed to the voters. There always exists an equilibrium where both candidates announce the median of the state that is most likely given the voters' signal. We refer to this as pandering. When the voters' signal is not too accurate then polarization is also possible. The reason why revelation is not possible in this case is that candidates are not uncertain about the belief of the voters after a deviation because they observe everything that voters observe. This makes it impossible to sustain revelation.

Following the equilibrium analysis of the two cases we compare them with respect to the welfare of the voters, more precisely with respect to each voter's ex ante expected utility. We show that when voters' information is sufficiently accurate then all voters are better off in Case 1, i.e., when candidates do not observe their signal before taking positions. Otherwise all voters are better off in Case 2. This result follows from the equilibrium analysis and the following Pareto ranking of strategy profiles: Revelation (convergence to the true median) dominates pandering which dominates polarization. This ranking of strategy profiles is quite intuitive. When the candidates converge on the true median voters can infer the true state and the candidates' positions are, from an ex ante perspective, optimal to the voters. When candidates pander voters cannot infer the true state but the candidates' choices of positions are at least responsive to the signal of the voters. In the polarization profile voters again cannot infer the true state and the candidates' positions do not reflect any of the information in the game.

With respect to welfare we also show that, within each case, all voters become better off when their signal becomes more informative. Finally we show the

interesting result that, in Case 1, increasing the quality of both candidates can actually make all voters worse off because it can change the equilibrium outcome from revelation to polarization.

Among the models of elections when politicians are better informed than the electorate, Schultz (1996) and Martinelli (2001) are the most immediately related to ours. They both consider a similar set-up, electoral competition with two candidates who simultaneously announce credible positions. In Schultz (1996) candidates are policy-motivated and fully informed about the state of the world. Voters are uninformed so they only receive information from the candidates' positions. There is revelation in (refined) equilibrium if at least one of the candidates has policy preferences that are sufficiently similar to the preferences of the median voter. In any revealing equilibrium there is convergence to the median policy of the true state of the world.

Martinelli (2001) considers a model where both candidates and voters receive private information about the state of the world but candidates are better informed than voters. The main result is that a revealing equilibrium always exists. If candidates are completely policy-motivated then they do not converge in revealing equilibria. But if office-motivation is sufficiently strong then there is convergence.

Our model is also related to the literature on candidate quality (valence). Among the contributions to this literature are Ansolabehere and Snyder (2000), Groseclose (2001) and Aragonés and Palfrey (2002, 2005). These papers all analyze models of electoral competition where candidates differ in quality such that if they announce sufficiently similar policy positions then each voter votes for the candidate of highest quality. There is no uncertainty about who the high quality candidate is. This is fundamentally different from our model where no candidate has an ex ante quality advantage because quality is state-dependent.

Krasa and Polborn (2009a, 2009b) analyze models of electoral competition where candidates' abilities are policy dependent (see also Schofield (2003) for a model with an additive, policy-dependent valence term). For example, some candidates may be better at running a small government while other candidates are better at running a big government. When candidates have different abilities it is shown that, even though candidates are completely office motivated, there will be policy divergence in equilibrium because candidates play to their own strengths. The idea of policy dependent candidate abilities is obviously related to the idea of state-dependent candidate quality that we present in this paper. However, the ideas are also clearly distinct. In our model voters' evaluation of candidates' characteristics does not directly depend on their policy positions. If, for example, voters know the true state and candidates' positions are close then the choice of the voters does not depend on where the candidates' positions are in the policy space.

The paper is organized as follows. In Section 2 we set up the model and in

Section 3 we consider strategies and beliefs and define our notion of equilibrium. Section 4 contains our equilibrium results for the two cases and we do welfare comparisons in Section 5. Finally, we discuss and conclude in Section 6.

2 The Model

We consider a one issue election. The policy space X is the real axis or some bounded interval. There are two purely office-motivated candidates, i.e., their only objective is to maximize the probability of winning. The candidates simultaneously announce credible policy positions before the election.

The electorate consists of an odd number of voters. The voters have utility functions over the policy space. These depend on the state of the world ω which can be either L or H . The policy utility function of voter i is

$$u_i(x|\omega) = -|x - x_i^*(\omega)|,$$

where $x_i^*(\omega)$ is the preferred policy of voter i in state ω . Each voter's preferred policy is further to the right in state H than in state L . More precisely we assume that, for each voter i ,

$$x_i^*(H) = x_i^*(L) + D,$$

where $D > 0$ is a constant. Thus the median voter is the same in the two states. His preferred policies are denoted $x_m^*(L)$ and $x_m^*(H)$.

Besides policy, voters also care about candidate quality which is state-dependent. One candidate, *Candidate L*, has a quality advantage in state L while the other candidate, *Candidate H*, has a quality advantage of the same size in state H . On top of this there is a symmetric stochastic element to each voter's candidate preference. These two features are modelled the following way. Suppose Candidate L has announced the policy x^L and that Candidate H has announced x^H . Then voter i 's utility if Candidate L is elected is

$$U_i((L, x^L)|\omega) = \left\{ \begin{array}{ll} u_i(x^L|L) + \gamma & \text{if } \omega = L \\ u_i(x^L|H) & \text{if } \omega = H \end{array} \right\} + \delta,$$

where $\gamma > 0$ is a parameter and, for some parameter $\sigma > 0$, δ is drawn from the uniform distribution on the interval $[-\frac{1}{2\sigma}, \frac{1}{2\sigma}]$. The realized value of δ is the same for all voters, independent of the state of the world, and unknown to the candidates when they announce positions. Voter i 's utility if Candidate H is elected is

$$U_i((H, x^H)|\omega) = \left\{ \begin{array}{ll} u_i(x^H|L) & \text{if } \omega = L \\ u_i(x^H|H) + \gamma & \text{if } \omega = H \end{array} \right\}.$$

Each voter votes for the candidate providing the highest expected utility. So if voter i believes that the probability of state L is μ_L then he votes for Candidate L if and only if

$$\mu_L(u_i(x^L|L)+\gamma+\delta)+(1-\mu_L)(u_i(x^L|H)+\delta) > \mu_L u_i(x^H|L)+(1-\mu_L)(u_i(x^H|H)+\gamma).$$

This is equivalent to

$$\delta > \mu_L(u_i(x^H|L) - u_i(x^L|L) - \gamma) + (1 - \mu_L)(u_i(x^H|H) - u_i(x^L|H) + \gamma).$$

Plugging in the policy utility function of the voter this inequality becomes

$$\begin{aligned} \delta > \mu_L(|x^L - x_i^*(L)| - |x^H - x_i^*(L)| - \gamma) \\ + (1 - \mu_L)(|x^L - x_i^*(H)| - |x^H - x_i^*(H)| + \gamma). \end{aligned} \quad (1)$$

With respect to information, both candidates are fully informed about the state of the world. The voters only receive a signal $\omega^V \in \{l, h\}$ which is distributed according to

$$\Pr(\omega^V = l|L) = \Pr(\omega^V = h|H) = \theta^V,$$

where $\frac{1}{2} < \theta^V < 1$. All voters receive the same signal and they all have the prior belief that each state is equally likely. Thus, if voters do Bayesian updating based on their signal then their belief about the state is simply given by

$$\Pr(L|\omega^V = l) = \Pr(H|\omega^V = h) = \theta^V.$$

We consider both the case where the signal of the voters is unknown to the candidates and the case where they know it. When we assume that the voters' signal is unknown to the candidates it should not be taken to mean that candidates do not receive the information that voters do. Because candidates are, of course, also voters. Instead it should be interpreted as a situation where candidates have to announce policy positions relatively early in the campaign and are uncertain about what information voters will receive between then and election day.

The timeline of the game where candidates do not know the signal of the voters when they take positions is:

1. The candidates observe the true state of the world;
2. The candidates simultaneously announce policy positions;
3. The voters receive the signal ω^V and the value of δ is realized;
4. The voters cast their votes;

5. The winning candidate enacts his announced position.

We refer to this situation as *Case 1*. If candidates know the signal of the voters when taking positions (*Case 2*) the timeline is as above except that voters (and thus also candidates) receive the signal ω^V in step 1.

Throughout the paper we assume that

$$\gamma + D < \frac{1}{2\sigma}.$$

This condition implies that if the distance between the candidates' positions is at most D then, no matter what voters believe about the state, each candidate has a strictly positive probability of winning the election. It simplifies our analysis considerably because it ensures that in all situations we need to consider the realization of δ matters. Alternatively we could have assumed that δ is drawn from some distribution with full support on \mathbb{R} , for example the normal distribution. But this would, we believe, only complicate the analysis of the model without adding substantial insight.

3 Strategies, Beliefs, and Equilibrium

In Case 1 a strategy for one of the candidates consists of a policy position for each state ω . Thus a strategy for Candidate i , $i = L, H$, can be written

$$x^i(\omega).$$

In Case 2 the candidates observe the signal of the voters before taking positions. Therefore a strategy for Candidate i in this case can be written

$$x^i(\omega, \omega^V).$$

As mentioned above, each voter votes for the candidate who provides the highest expected utility. To calculate the expected utility provided by each candidate each voter forms a belief about the true state of the world. This belief can depend on everything the voter observes, i.e., the positions of the candidates and the signal ω^V . We will make the simplifying assumption that all voters share the same belief. The belief about the probability of state L is written

$$\mu_L(x^L, x^H, \omega^V),$$

where x^L and x^H are the observed policy positions of the two candidates.

When all voters share the same belief function we have the standard result that the median voter decides the outcome of the election.

Lemma 3.1 (The Median Voter Decides the Outcome)

Suppose that, given the candidates' positions, the voters' signal, and the realization of δ , the median voter strictly prefers Candidate L (H). Then a strict majority of voters strictly prefers Candidate L (H).

Proof. Let x^L and x^H be the positions of the candidates and let μ_L be the belief of the voters given these positions and the voters' signal. Suppose the median voter strictly prefers Candidate L (the other case is analogous), i.e.,

$$\delta > \mu_L(|x^L - x_m^*(L)| - |x^H - x_m^*(L)| - \gamma) + (1 - \mu_L)(|x^L - x_m^*(H)| - |x^H - x_m^*(H)| + \gamma).$$

We then have to show that, for each voter i in a strict majority,

$$\delta > \mu_L(|x^L - x_i^*(L)| - |x^H - x_i^*(L)| - \gamma) + (1 - \mu_L)(|x^L - x_i^*(H)| - |x^H - x_i^*(H)| + \gamma).$$

Suppose $x^L \leq x^H$ (the other case is analogous). It suffices to show that the inequality above holds for all voters i with $x_i^*(L) \leq x_m^*(L)$. This is the case if

$$|x^L - x_i^*(L)| - |x^H - x_i^*(L)| \leq |x^L - x_m^*(L)| - |x^H - x_m^*(L)|$$

and

$$|x^L - x_i^*(H)| - |x^H - x_i^*(H)| \leq |x^L - x_m^*(H)| - |x^H - x_m^*(H)|$$

for all such voters, which is straightforward to verify. \square

Now consider the objective of Candidate L given Candidate H 's strategy and the belief function of the voters. First we consider Case 1 where candidates do not know the signal of the voters when they take positions. The strategy of Candidate H is $x^H(\omega)$. Suppose Candidate L takes the position x . Then, by the median voter result above and inequality (1) from the previous section, his probability of winning the election in state ω can be written

$$\begin{aligned} \sum_{\omega^V=l,h} \Pr(\omega^V|\omega) \Pr_{\delta}[\delta > \mu_L(x, x^H(\omega), \omega^V)(|x - x_m^*(L)| - |x^H(\omega) - x_m^*(L)| - \gamma) \\ + (1 - \mu_L(x, x^H(\omega), \omega^V))(|x - x_m^*(H)| - |x^H(\omega) - x_m^*(H)| + \gamma)]. \end{aligned}$$

This is simply the probability of receiving the vote of the median voter with respect to the distribution of δ and the distribution of ω^V given ω . The objective of Candidate L in state ω then is to maximize this expression with respect to x .

In Case 2 candidates know the signal of the voters when they take positions so the strategy of Candidate H is written $x^H(\omega, \omega^V)$. Again let x denote the position of Candidate L . Then his probability of winning the election given ω and ω^V is

$$\begin{aligned} \Pr_{\delta}[\delta > \mu_L(x, x^H(\omega, \omega^V), \omega^V)(|x - x_m^*(L)| - |x^H(\omega, \omega^V) - x_m^*(L)| - \gamma) \\ + (1 - \mu_L(x, x^H(\omega, \omega^V), \omega^V))(|x - x_m^*(H)| - |x^H(\omega, \omega^V) - x_m^*(H)| + \gamma)]. \end{aligned}$$

His objective is, of course, to maximize this probability with respect to x . Given the objective of Candidate L it is, in both cases, obvious how to write out the objective of Candidate H , so we will not do it.

Then we are ready to define our notion of equilibrium. We use the standard concept of Perfect Bayesian Equilibrium (with the additional assumption that all voters have the same belief function). We first consider Case 1.

Definition 3.2 (Equilibrium, Case 1)

An equilibrium consists of candidate strategies,

$$\hat{x}^L(\omega) \text{ and } \hat{x}^H(\omega),$$

and a voter belief function about the probability of state L ,

$$\hat{\mu}_L(x^L, x^H, \omega^V),$$

such that the following two conditions hold.

1. In each state of the world, each candidate's position maximizes his probability of winning given the other candidates position, the belief function of the voters, the distribution of the voters' signal conditional on the state, and the distribution of δ .
2. The belief function is consistent with Bayes' rule on the equilibrium path. I.e., if $\hat{x}^L(L) \neq \hat{x}^L(H)$ or $\hat{x}^H(L) \neq \hat{x}^H(H)$ then, for both values of ω^V ,

$$\hat{\mu}_L(\hat{x}^L(L), \hat{x}^H(L), \omega^V) = 1 \text{ and } \hat{\mu}_L(\hat{x}^L(H), \hat{x}^H(H), \omega^V) = 0.$$

If $\hat{x}^L(L) = \hat{x}^L(H) = \hat{x}^L$ and $\hat{x}^H(L) = \hat{x}^H(H) = \hat{x}^H$ then

$$\hat{\mu}_L(\hat{x}^L, \hat{x}^H, l) = \theta^V \text{ and } \hat{\mu}_L(\hat{x}^L, \hat{x}^H, h) = 1 - \theta^V.$$

An equilibrium where voters can infer the true state of the world, i.e., an equilibrium where at least one candidate takes different policy positions in the two states, is called a *revealing equilibrium*. An equilibrium where voters cannot infer the true state and thus have to rely on their own signal is called a *non-revealing equilibrium*.

For Case 2 the definition of equilibrium is analogous, so we will not write it out in detail. Of course the strategies of the candidates can depend on both ω and ω^V and the candidates do not have to consider the distribution of ω^V given ω . We distinguish between three types of equilibria. In a *fully revealing equilibrium* there is always (i.e., for both values of ω^V) at least one candidate who reveals the state. In a *partially revealing equilibrium* the state is revealed for one value of the voters signal but not for the other. For example, suppose $\hat{x}^i(L, l) \neq \hat{x}^i(H, l)$ for at least one i and $\hat{x}^i(L, h) = \hat{x}^i(H, h)$ for both i . Then voters can infer the true state when $\omega^V = l$ but they cannot when $\omega^V = h$. Finally, in a *non-revealing equilibrium* the state is not revealed to the voters for any value of ω^V .

4 Equilibrium Analysis

4.1 Case 1

We first consider revealing equilibria. We will introduce a refinement condition that puts restrictions on out-of-equilibrium beliefs in such equilibria. The refinement condition is based on the concept of unprejudiced beliefs introduced by Bagwell and Ramey (1991) in a two-sender signalling game (a limit pricing model with two incumbents). Unprejudiced beliefs has been used in a model of electoral competition by Schultz (1996).

Formulated in terms of our model, the idea behind the concept of unprejudiced beliefs is the following. Suppose $\hat{x}^L(\omega)$ and $\hat{x}^H(\omega)$ are the candidate strategies in some revealing equilibrium and that voters observe some non-equilibrium pair of positions (x^L, x^H) . Then voters consider, for each state of the world, whether it takes only one or both candidates to deviate in order to generate (x^L, x^H) . If this is different for the two states then voters will assume that the true state is the one for which it requires only one deviation to generate (x^L, x^H) . For example, if the non-equilibrium positions satisfy $x^L = \hat{x}^L(L)$, $x^L \neq \hat{x}^L(H)$, and $x^H \neq \hat{x}^H(H)$ then voters will assume that $\omega = L$. An equivalent formulation of the concept is that if one candidate's equilibrium strategy is revealing and the other candidate deviates to an out-of-equilibrium position then voters will believe the non-deviating candidate.

Here we will not assume that all unilateral out-of-equilibrium deviations from an equilibrium where the non-deviating candidate's strategy is revealing will leave the beliefs of the voters unchanged. We will instead make the substantially weaker assumption that if the deviation is sufficiently small then it will not change voters' beliefs, i.e., they will still believe the non-deviating candidate. This weakening of the concept of unprejudiced beliefs was suggested by Yehezkel (2006) and is supported by recent experimental evidence on oligopoly limit pricing (Müller, Spiegel, and Yehezkel 2009). We give the formal definition below (we use the term ε -unprejudiced).

Definition 4.1 (ε -Unprejudiced Revealing Equilibrium)

Consider a revealing equilibrium $(\hat{x}^L(\omega), \hat{x}^H(\omega), \hat{\mu}_L(x^L, x^H, \omega^V))$. It is ε -unprejudiced if the following conditions are satisfied.

1. Suppose $\hat{x}^L(L) \neq \hat{x}^L(H)$. Then there exists an $\varepsilon > 0$ such that

$$\hat{\mu}_L(\hat{x}^L(L), x, l) = \hat{\mu}_L(\hat{x}^L(L), x, h) = 1 \text{ if } x \neq \hat{x}^H(H) \text{ and } |\hat{x}^H(L) - x| < \varepsilon$$

and

$$\hat{\mu}_L(\hat{x}^L(H), x, l) = \hat{\mu}_L(\hat{x}^L(H), x, h) = 0 \text{ if } x \neq \hat{x}^H(L) \text{ and } |\hat{x}^H(H) - x| < \varepsilon.$$

2. Suppose $\hat{x}^H(L) \neq \hat{x}^H(H)$. Then there exists an $\varepsilon > 0$ such that

$$\hat{\mu}_L(x, \hat{x}^H(L), l) = \hat{\mu}_L(x, \hat{x}^H(L), h) = 1 \text{ if } x \neq \hat{x}^L(H) \text{ and } |\hat{x}^L(L) - x| < \varepsilon$$

and

$$\hat{\mu}_L(x, \hat{x}^H(H), l) = \hat{\mu}_L(x, \hat{x}^H(H), h) = 0 \text{ if } x \neq \hat{x}^L(L) \text{ and } |\hat{x}^L(H) - x| < \varepsilon.$$

It could be argued that requiring the belief functions of revealing equilibria to be unprejudiced for some deviations is a stronger assumption here than in standard multi-sender signalling games. Because in our model the receivers (the voters) have some information of their own. If voters still believe the non-deviating candidate (with a revealing strategy) after a deviation it means that they continue to ignore their own signal. This is probably not realistic for large deviations. Indeed, the experiments in Müller, Spiegel, and Yehezkel (2009) provide evidence that large out-of-equilibrium deviations make receivers change their beliefs even in standard signalling games. However, as long as we restrict the condition to (arbitrarily) small deviations we think that it is a realistic refinement condition in our setting.

Our first result shows that in any ε -unprejudiced revealing equilibrium the candidates converge on the median position of the true state.

Proposition 4.2 (ε -Unprejudiced Revealing Equilibria: Strategies)

In any ε -unprejudiced revealing equilibrium the candidate strategies are given by

$$\hat{x}^L(L) = \hat{x}^H(L) = x_m^*(L) \text{ and } \hat{x}^L(H) = \hat{x}^H(H) = x_m^*(H).$$

Proof. Let $\hat{x}^L(\omega)$, $\hat{x}^H(\omega)$ be the candidate strategies in an ε -unprejudiced revealing equilibrium. At least one of the candidates must announce different policies in the two states. Suppose $\hat{x}^L(L) \neq \hat{x}^L(H)$ (the other case is analogous). Assume that $\hat{x}^H(L) \neq x_m^*(L)$ and consider a state L -deviation by Candidate H to a position $x \neq \hat{x}^H(H)$ that is slightly closer to $x_m^*(L)$. This deviation will increase the median voter's utility of Candidate H being elected in state L . And since a sufficiently small out-of-equilibrium deviation does not change the belief of the voters (they will still believe with certainty that the state is L) it follows that candidate H 's probability of winning is higher after the deviation, which is a contradiction. Therefore we must have $\hat{x}^H(L) = x_m^*(L)$. Similarly we get $\hat{x}^H(H) = x_m^*(H)$. And then we can use the same argument for Candidate L to get $\hat{x}^L(L) = x_m^*(L)$ and $\hat{x}^L(H) = x_m^*(H)$. \square

Our next step is to find the set of parameter values for which an ε -unprejudiced revealing equilibrium exists. The following result shows that we have existence if and only if the voters' signal is sufficiently informative. The proof is in Appendix A.

Proposition 4.3 (ε -Unprejudiced Revealing Equilibria: Existence)

There exists an ε -unprejudiced revealing equilibrium if and only if

$$\theta^V \geq \theta_R^*,$$

where

$$\theta_R^* = \frac{1}{2} \left(1 + \frac{\gamma}{\gamma + D} \right).$$

The reasoning behind the result is as follows. It is easy to define beliefs such that it is never profitable for a candidate to deviate to a position $x \neq x_m^*(L), x_m^*(H)$ (simply let the belief of the voters be unchanged after such a deviation). Thus we only have to consider deviations to the wrong median, i.e., to $x_m^*(H)$ when $\omega = L$ and to $x_m^*(L)$ when $\omega = H$. So, loosely speaking, we have an equilibrium if neither candidate can profitably lie to the electorate. It is easy to define the belief function such that it is never profitable for the high quality candidate (Candidate L when $\omega = L$, Candidate H when $\omega = H$) to lie. So we have an equilibrium if and only if we can define a belief function such that, in each state, it is not profitable for the low quality candidate to lie by announcing the wrong median. This can only be done for sufficiently high values of θ^V . Because otherwise the risk of "getting caught" by the voters and ending up with both a quality and a policy disadvantage is too small.

θ_R^* is obviously increasing in γ . So if the quality parameter γ increases then the electorate has to be better informed in order to make the candidates reveal their information. For higher γ it is relatively more costly (in terms of probability of winning) for the low quality candidate to reveal the state. So when γ increases the new cut-off value of θ^V must make it more costly for the low quality candidate not to reveal, i.e., it must be higher. We also see that when the difference in candidate quality vanishes then there exists a revealing equilibrium no matter how little information the electorate has ($\lim_{\gamma \rightarrow 0} \theta_R^* = \frac{1}{2}$).

Also note that θ_R^* is decreasing in D , the distance between $x_m^*(H)$ and $x_m^*(L)$. So when the median positions of the two states are further apart then a revealing equilibrium exist for less informed electorates. The reason is that a higher D makes it relatively more costly to lie for the disadvantaged candidate. The good news from this observation is that when the state of the world really matters for policy choice (i.e., D is high) then it takes less voter information to make the candidates reveal their information by converging to the median of the true state.

We now move on to consider non-revealing equilibria. In non-revealing equilibria each candidate announces the same position in both states of the world. Thus the equilibrium strategies for Candidate L and H can simply be written \hat{x}^L and \hat{x}^H .

We first consider equilibria with no restrictions on voters' out-of-equilibrium beliefs. We limit our attention to equilibria where the position of each candidate is between the median positions of the two states of the world. This is justified by the following observation. Suppose $(\hat{x}^L, \hat{x}^H) \notin [x_m^*(L), x_m^*(H)]^2$ is the strategy profile of a non-revealing equilibrium. Then so is the strategy profile where, for each i , \hat{x}^i is replaced by $x_m^*(L)$ if $\hat{x}^i < x_m^*(L)$ and by $x_m^*(H)$ if $x_m^*(H) < \hat{x}^i$. Thus, if there exists a non-revealing equilibrium where at least one candidate is not positioned between the two medians then there exists a corresponding equilibrium with strategy profile in $[x_m^*(L), x_m^*(H)]^2$. A proof of this observation is given in Appendix A.

In the following proposition we find all non-revealing equilibria where the position of each candidate is between the medians of the two states of the world. The proof is in Appendix A.

Proposition 4.4 (Non-Revealing Equilibria: Strategies and Existence)

Let $(\hat{x}^L, \hat{x}^H) \in [x_m^*(L), x_m^*(H)]^2$. Then the following statements hold.

1. If $\max\{\hat{x}^L - x_m^*(L), x_m^*(H) - \hat{x}^H\} < \gamma$ then (\hat{x}^L, \hat{x}^H) is the strategy profile of a non-revealing equilibrium if and only if

$$\theta^V \leq \frac{1}{2} \left(1 + \sqrt{\frac{\gamma - \max\{\hat{x}^L - x_m^*(L), x_m^*(H) - \hat{x}^H\}}{\gamma + (\hat{x}^H - \hat{x}^L)}} \right).$$

2. If $\max\{\hat{x}^L - x_m^*(L), x_m^*(H) - \hat{x}^H\} \geq \gamma$ then (\hat{x}^L, \hat{x}^H) is the strategy profile of a non-revealing equilibrium if and only if

$$\hat{x}^L - \hat{x}^H = \gamma.$$

Briefly described, the arguments of the proof are as follows. If $\max\{\hat{x}^L - x_m^*(L), x_m^*(H) - \hat{x}^H\} < \gamma$ then any deviation x by Candidate L (H) gives the lowest probability of winning if we let $\hat{\mu}_L(x, \hat{x}^H, \omega^V) = 0$ ($\hat{\mu}_L(\hat{x}^L, x, \omega^V) = 1$) for both values of ω^V . Therefore (\hat{x}^L, \hat{x}^H) is an equilibrium strategy profile precisely if candidate L cannot profitably deviate to $x_m^*(H)$ when $\omega = H$ and $\hat{\mu}_L(x_m^*(H), \hat{x}^H, \omega^V) = 0$ and candidate H cannot profitably deviate to $x_m^*(L)$ when $\omega = L$ and $\hat{\mu}_L(\hat{x}^L, x_m^*(L), \omega^V) = 1$. These conditions lead directly to the cut-off value for θ^V . If $\max\{\hat{x}^L - x_m^*(L), x_m^*(H) - \hat{x}^H\} \geq \gamma$ there is at least one candidate who can always win with a probability equal or arbitrarily close to $\frac{1}{2}$ by deviating (no matter how we specify out-of-equilibrium beliefs). Thus it must be the case that this candidate always wins with a probability of at least $\frac{1}{2}$ in equilibrium. This is only possible if $\hat{x}^L - \hat{x}^H = \gamma$.

An immediate consequence of Proposition 4.4 is that a non-revealing equilibrium with candidate positions in $[x_m^*(L), x_m^*(H)]$ always exists. If $D < \gamma$ then it follows from the first statement that there exists an equilibrium with $\hat{x}^L = x_m^*(H)$ and $\hat{x}^H = x_m^*(L)$ for all values of θ^V . If $\gamma \leq D$ then it follows from the second statement that, for all values of θ^V , there exists an equilibrium with $\hat{x}^L - \hat{x}^H = \gamma$.

The abundance of non-revealing equilibria makes it natural to ask if some of them can be eliminated by a suitable refinement condition. In signalling games the most commonly used refinement condition is the Intuitive Criterion (Cho and Kreps 1987). In games where there are only two sender-types (t_1 and t_2) the content of the criterion is that if a deviation from equilibrium could be profitable only for t_i then, upon observing this deviation, receivers should believe with certainty that the sender is of this type. To make precise what this means in our model, consider a non-revealing equilibrium $\hat{x}^L, \hat{x}^H, \hat{\mu}_L(x^L, x^H, \omega^V)$ and a deviation by Candidate L to some x . Suppose that we can make the deviation profitable (by changing the out-of-equilibrium beliefs) if and only if $\omega = L$ ($\omega = H$). Then the equilibrium satisfies the intuitive criterion if $\hat{\mu}_L(x, \hat{x}^H, \omega^V) = 1$ ($\hat{\mu}_L(x, \hat{x}^H, \omega^V) = 0$). Of course, analogous restrictions should hold in out-of-equilibrium situations where Candidate H deviates.

It turns out that the Intuitive Criterion does not eliminate any of the non-revealing equilibria from Proposition 4.4. The argument is rather straightforward. Consider a deviation by candidate L to some position x (the argument for deviations by candidate H is completely analogous). The maximum probability of winning that candidate L can achieve by this deviation when we allow out-of-equilibrium beliefs to be changed is independent of the state (because we get the maximum probability of winning by letting $\mu_L(x, \hat{x}^H, \omega^V) = 0$ or $\mu_L(x, \hat{x}^H, \omega^V) = 1$ for both values of ω^V and thus the conditional distribution of ω^V given the state does not matter). In all equilibria candidate L wins with probability $p \geq \frac{1}{2}$ in state L and probability $1 - p$ in state H . So if x could be a profitable deviation in state L then the same is true in state H . Thus the equilibrium belief function does not violate the Intuitive Criterion if, for all $x \neq \hat{x}^L$, $\hat{\mu}_L(x, \hat{x}^H, \omega^V) = 0$. All equilibrium strategies considered in the first statement of the proposition are supported by belief functions satisfying this conditions. This is not true for the equilibrium strategies considered in the second part of the proposition. But in this case each candidate's equilibrium probability of winning is $\frac{1}{2}$ in both states and thus the Intuitive Criterion does not put any restrictions at all on out-of-equilibrium beliefs.

One way of eliminating many of the non-revealing equilibria is to impose a monotonicity condition on the voter belief function. We will assume that $\hat{\mu}_L(x^L, x^H, \omega^V)$ is, for both values of ω^V , weakly decreasing with respect to both x^L and x^H . Thus we assume that a move to the right by one of the candidates will not make voters believe that state L is more likely. There is no directly state-dependent cost for the candidates that can justify this assumption. Nevertheless, since each voter's

preferred policy is further to the right in state H than in state L and candidates are completely office-motivated it does seem intuitively reasonable to assume that a candidate move to the right will not make voters believe that state L is more likely¹. It is important to note that restricting attention to monotone equilibria (i.e., equilibria with monotone belief functions) does not change our result on existence of ε -unprejudiced revealing equilibria. More precisely, the conclusion from Proposition 4.3 still holds if we also require revealing equilibria to be monotone.

In the following proposition we find the candidate positions that are possible in monotone non-revealing equilibria.

Proposition 4.5 (Monotone Non-Revealing Equilibria: Strategies)

The candidate positions in any monotone non-revealing equilibrium satisfy

$$\hat{x}^L \leq x_m^*(L) \quad \text{and} \quad x_m^*(H) \leq \hat{x}^H.$$

Proof. Suppose $\hat{x}^L > x_m^*(L)$. Consider a deviation by Candidate L to $x_m^*(L)$. This deviation will increase the median voter's utility of Candidate L being elected at least as much in state L as it will decrease it in state H . Furthermore, the deviation will, by monotonicity of the belief function, not make voters believe that state L is less likely. Therefore it is easy to see that the deviation is profitable in state L . Thus we must have $\hat{x}^L \leq x_m^*(L)$ and by symmetry it follows that $x_m^*(H) \leq \hat{x}^H$. \square

When the monotonicity condition is imposed on non-revealing equilibria we get that such equilibria exist if and only if the accuracy of the voters' signal is below some cut-off value. From the first statement in Proposition 4.4 we already know the cut-off value for existence of a non-revealing equilibrium with $\hat{x}^L = x_m^*(L)$ and $\hat{x}^H = x_m^*(H)$. And the belief function used to support these positions as an equilibrium can easily be modified to satisfy monotonicity in such a way that the arguments in the proof still hold. Simply change the belief of the voters to be given by Bayesian updating on their signal if Candidate H deviates to the right or Candidate L deviates to the left. Furthermore, using Proposition 4.5 it is easy to see that if some pair of positions is the strategy profile of a monotone non-revealing equilibrium then so is $(x_m^*(L), x_m^*(H))$. Thus we have the following corollary.

¹It could be argued that since the median voter decides the outcome of the election we should instead use the following alternative monotonicity assumption: If one candidate moves to a position that is closer to $x_m^*(H)$ and not closer to $x_m^*(L)$ then this should not make voters believe that state L is more likely (and vice versa). It turns out that using this alternative condition would not change our results. And since it requires more voter sophistication (all voters have to know the exact location of the median positions) we find it somewhat less intuitively appealing.

Corollary 4.6 (Monotone Non-Revealing Equilibria: Existence)

There exists a monotone non-revealing equilibrium if and only if

$$\theta^V \leq \theta_N^*,$$

where

$$\theta_N^* = \frac{1}{2} \left(1 + \sqrt{\frac{\gamma}{\gamma + D}} \right).$$

For any $\theta^V \leq \theta_N^*$ there exists a monotone non-revealing equilibrium with

$$\hat{x}^L = x_m^*(L) \text{ and } \hat{x}^H = x_m^*(H).$$

From Proposition 4.3 and Corollary 4.6 it follows that, for all parameter values, either an ε -unprejudiced (and monotone) revealing equilibrium or a monotone non-revealing equilibrium exists. We also see that for some parameter values ($\theta_R^* \leq \theta^V \leq \theta_N^*$) both types of equilibria exist.

4.2 Case 2

We first consider equilibria that are fully or partially revealing, i.e., where the state is revealed to the voters for at least one value of their signal. As in Case 1 we restrict attention to equilibria where, if one candidate's strategy reveals the state then a small out-of-equilibrium deviation by the other candidate does not change voters' beliefs (ε -unprejudiced equilibria). Again it is fairly straightforward to show that if the state is revealed to the voters in equilibrium then each candidate's position must be the median of the true state (see Proposition 4.2). So if $\hat{x}^i(L, \omega^V) \neq \hat{x}^i(H, \omega^V)$ for at least one i then we must have

$$\hat{x}^L(L, \omega^V) = \hat{x}^H(L, \omega^V) = x_m^*(L) \quad \text{and} \quad \hat{x}^L(H, \omega^V) = \hat{x}^H(H, \omega^V) = x_m^*(H).$$

Thus, in any ε -unprejudiced fully revealing equilibrium the candidates always converge on the median of the true state.

It turns out that neither fully nor partially revealing ε -unprejudiced equilibria exist. Suppose we have an equilibrium which is revealing for $\omega^V = l$. Then, for this value of the voters' signal, both candidates must be at $x_m^*(L)$ in state L and at $x_m^*(H)$ in state H . Two necessary conditions for equilibrium are that candidate H cannot profitably deviate to $x_m^*(H)$ in state L and that candidate L cannot profitably deviate to $x_m^*(L)$ in state H . It is easy to see that the first condition implies $\hat{\mu}_L(x_m^*(L), x_m^*(H), l) > \frac{1}{2}$ and that the second condition implies $\hat{\mu}_L(x_m^*(L), x_m^*(H), l) < \frac{1}{2}$, which is a contradiction.

We then turn attention to non-revealing equilibria. Strategies in non-revealing equilibria can depend only on the value of the voters' signal. Thus equilibrium strategies for the two candidates can be written $\hat{x}^L(\omega^V)$ and $\hat{x}^H(\omega^V)$.

Analogously to Case 1 we can find all non-revealing equilibria where positions are between the two medians (restricting attention to such equilibria is justified by the same argument as in Case 1, see p. 14). Since we are primarily interested in monotone non-revealing equilibria we defer the result to Appendix B.

Imposing the same condition of monotonicity as in Case 1 we get that in any non-revealing equilibrium we must have

$$\hat{x}^L(l) \leq x_m^*(L) \quad \text{and} \quad x_m^*(H) \leq \hat{x}^H(h).$$

Because if, for example, $\hat{x}^L(l) > x_m^*(L)$ then candidate L could win with a higher probability by deviating to $x_m^*(L)$ when $\omega^V = l$ (see the proof of Proposition 4.5 for more details, the argument is analogous).

Then we are ready to formulate our main result on monotone non-revealing equilibria. The proof is in Appendix A.

Proposition 4.7 (Mon. Non-Rev. Equilibria: Strategies and Existence)

1. $((\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h))) \in [x_m^*(L), x_m^*(H)]^4$ is the strategy profile of a monotone non-revealing equilibrium if and only if

$$\hat{x}^L(l) = x_m^*(L) \quad \text{and} \quad \hat{x}^H(h) = x_m^*(H)$$

and

$$\theta^V \leq \frac{1}{2} \left(1 + \frac{\gamma}{\gamma + \max\{\hat{x}^H(l) - x_m^*(L), x_m^*(H) - \hat{x}^L(h)\}} \right).$$

2. If $((\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h))) \notin [x_m^*(L), x_m^*(H)]^4$ is the strategy profile of a monotone non-revealing equilibrium then so is the profile where, for each i and each value of ω^V , $\hat{x}^i(\omega^V)$ is replaced by $x_m^*(L)$ if $\hat{x}^i(\omega^V) < x_m^*(L)$ and by $x_m^*(H)$ if $x_m^*(H) < \hat{x}^i(\omega^V)$.

From the result we immediately see that there always exists a monotone non-revealing equilibrium with

$$\hat{x}^L(l) = \hat{x}^H(l) = x_m^*(L) \quad \text{and} \quad \hat{x}^L(h) = \hat{x}^H(h) = x_m^*(H),$$

i.e., an equilibrium where the candidates converge on the median that is most likely given the voters' signal. We refer to this type of candidate behavior as *pandering*. When θ^V is not too high then the candidate who is most likely to be the low quality candidate given the voters' signal (Candidate L when $\omega^V = h$, Candidate H when $\omega^V = l$) need not be close to the most likely median. For example, the strategies

$$\hat{x}^L(l) = \hat{x}^L(h) = x_m^*(L) \quad \text{and} \quad \hat{x}^H(l) = \hat{x}^H(h) = x_m^*(H)$$

are possible in monotone equilibrium if

$$\theta^V \leq \frac{1}{2} \left(1 + \frac{\gamma}{\gamma + D} \right).$$

So when the accuracy of the voters' signal is below this cut-off value both pandering and (full) polarization is possible.

5 Welfare

In this section we first do a welfare comparison of different strategy profiles. A strategy profile consists of a position for each candidate for each combination of the state and the signal of the voters. Obviously, only strategy profiles where positions do not depend on ω^V are relevant for Case 1. We will compare the profiles with respect to the *ex ante expected utility* that each voter obtains. When comparing the strategy profiles we will assume that the belief of the voters is given by Bayesian updating and that they vote based on expected utility. So in our comparison we assume that voters behave as they would if the strategy profile was part of an equilibrium. We use the welfare comparison of strategy profiles to do a welfare comparison of Case 1 and 2. More precisely, we compare the (refined) equilibria of the two cases for fixed parameter values. Furthermore, we show that, within each case, more voter information is welfare improving. Finally we discuss the welfare effect of a change in γ .

We focus on the three main types of candidate equilibrium behavior that we have encountered: Revelation, polarization, and pandering. Let us briefly recall what we mean by these three terms. Revelation means that, for both values of the voters' signal ω^V , the strategy profile is given by

$$\hat{x}^L(L, \omega^V) = \hat{x}^H(L, \omega^V) = x_m^*(L) \text{ and } \hat{x}^L(H, \omega^V) = \hat{x}^H(H, \omega^V) = x_m^*(H).$$

Polarization means that, for both values of ω^V ,

$$\hat{x}^L(L, \omega^V) = \hat{x}^L(H, \omega^V) = x_m^*(L) \text{ and } \hat{x}^H(L, \omega^V) = \hat{x}^H(H, \omega^V) = x_m^*(H).$$

Finally, pandering means that, for both states ω ,

$$\hat{x}^L(\omega, l) = \hat{x}^H(\omega, l) = x_m^*(L) \text{ and } \hat{x}^L(\omega, h) = \hat{x}^H(\omega, h) = x_m^*(H).$$

The voting decisions of the voters, and thus the outcome of the election, depend on their beliefs about the state (we assume that they maximize expected utility given their beliefs). Therefore, in order to calculate the ex ante expected utility of each of the three strategy profiles for each voter we must specify their beliefs. As

mentioned above, we assume that voters are Bayesians. Thus they learn the state if the strategy profile is revealing, otherwise (i.e., for the polarization and pandering profiles) they form their belief based on the value of ω^V . This is of course identical to what the beliefs would be in equilibrium. Therefore our comparison of strategy profiles makes it straightforward to do a comparison of equilibria.

With the above assumption on voters' beliefs, a strategy profile s defines an outcome map $o_s : (\omega, \omega^V, \delta) \mapsto (w, x^w)$, where $w \in \{L, H\}$ is the winning candidate and x^w is the policy position of this candidate. For example, if $\hat{x}^L(L, l) = x_m^*(L)$ in the strategy profile s and Candidate L wins the election when $\omega = L$, $\omega^V = l$, and $\delta = 0$ then we have $o_s(L, l, 0) = (L, x_m^*(L))$. Remember that $U_i((j, x^j)|\omega)$ denotes the utility of voter i if candidate j with position x^j is elected and the state is ω . Given a strategy profile s the utility of voter i in state ω as a function of ω^V and δ can then be written $U_i(o_s(\omega, \omega^V, \delta)|\omega)$. So voter i 's *ex ante expected utility* of the strategy profile s is

$$E_{\delta, (\omega, \omega^V)}[U_i(o_s(\omega, \omega^V, \delta)|\omega)] = \int_{-\frac{1}{2\sigma}}^{\frac{1}{2\sigma}} \left[\sum_{(\omega, \omega^V)} \Pr(\omega, \omega^V) U_i(o_s(\omega, \omega^V, \delta)|\omega) \right] \sigma d\delta,$$

where $\Pr(\omega, \omega^V)$ is the ex-ante probability of the state-signal pair (ω, ω^V) . Note that, since the prior probability of each state is $\frac{1}{2}$, we have

$$\Pr(L, l) = \Pr(H, h) = \frac{1}{2}\theta^V$$

and

$$\Pr(L, h) = \Pr(H, l) = \frac{1}{2}(1 - \theta^V).$$

Now we are ready to compare strategy profiles with respect to the ex ante expected utility of the voters. We make the comparison both with respect to policy utility only and with respect to total utility. We let d_i denote the distance of voter i from the median voter, i.e.,

$$d_i = |x_i^*(L) - x_m^*(L)| = |x_i^*(H) - x_m^*(H)|.$$

The proof of the result is in Appendix A.

Proposition 5.1 (Welfare Comparison of Strategy Profiles)

1. *With respect to ex ante expected policy utility (and for fixed parameter values), the following ranking of strategy profiles holds for all voters:*

$$\text{Revelation} \succeq \text{Pandering} \succeq \text{Polarization}.$$

For voters with $d_i < D$ the ranking is strict, for voters with $d_i \geq D$ the three strategy profiles all give the same ex ante expected policy utility.

2. With respect to ex ante expected total utility (and for fixed parameter values), the following ranking of strategy profiles holds for all voters:

Revelation \succ Pandering \succ Polarization.

From the result we see that, with respect to both policy and total utility, there is a clear Pareto ranking of the three types of candidate behavior. Revelation dominates pandering which dominates polarization. This ranking is quite intuitive. In the revelation profile voters infer the true state and both candidates are positioned at the true median. When candidates pander voters cannot infer the true state because the candidates converge on the most likely median given the value of ω^V . Therefore it is not surprising that, from an ex ante perspective, all voters are better off with revelation than with pandering². When candidates are polarized voters have the same information (the value of their own signal) as when they are pandering. But the candidates' positions do not reflect this information, each candidate always announces the same position. This is bad for the voters and therefore polarization is dominated by pandering.

We can use the results above to do a welfare comparison of Case 1 and 2. Is it optimal for the voters that their information is not known by the candidates when they take positions or is it the other way around? We do this welfare comparison the following way. First of all we restrict attention to refined equilibria where all positions are between the two medians. Furthermore, we disregard the asymmetric equilibria of Case 2 where the distance between $\hat{x}^L(h)$ and $x_m^*(H)$ is different from the distance between $\hat{x}^H(l)$ and $x_m^*(L)$ ³. Then, with these restrictions, for each case and each set of parameter values select the equilibrium that is optimal with respect to the ex ante expected total utility of each voter. The outcome of this selection is presented in the following table.

| | $\theta^V \geq \theta_R^*$ | $\theta^V < \theta_R^*$ |
|--------|----------------------------|-------------------------|
| Case 1 | Revelation | Polarization |
| Case 2 | Pandering | Pandering |

²Some voters may of course prefer pandering over revelation for specific values of ω and ω^V . For example, voters with $x_i^*(L), x_i^*(H) > x_m^*(H)$ will obviously get a higher policy utility from pandering than from revelation when $\omega = L$ and $\omega^V = h$.

³These equilibria could be included in the welfare comparison of the two cases in the following natural way. For each asymmetric refined equilibrium consider a lottery where there is probability $\frac{1}{2}$ of ending up in this equilibrium and probability $\frac{1}{2}$ of ending up in its "mirror image" (i.e., the equilibrium where the distance between the position of Candidate L (H) and $x_{m_H}^*$ ($x_{m_L}^*$) when $\omega^V = h$ (l) is equal to the distance between the position of Candidate H (L) and $x_{m_L}^*$ ($x_{m_H}^*$) when $\omega^V = l$ (h) in the original equilibrium). Calculate each voter's expected utility of this lottery. Finally, compare with symmetric equilibria and similar lotteries between other asymmetric equilibria. With this way of including asymmetric equilibria in the welfare comparison all results remain the same.

Note that equilibria in Case 2 that involve some polarization and some pandering are, like polarization, dominated by pandering. The proof of this claim is similar to the proof that pandering dominates polarization. We then compare the two cases by comparing the optimal equilibria for fixed parameter values. We see that if voters are sufficiently well informed about the state of the world ($\theta^V \geq \theta_R^*$) then Case 1 is optimal. But if they are not then Case 2 is optimal. Note that this result remains the same if we compare the cases with respect to ex ante expected policy utility (except for the fact that with respect to policy utility some voters are indifferent between the two cases so we have a weak rather than strong Pareto ranking). Also note that if we instead of comparing optimal equilibria say that Case 1 is better than Case 2 if any equilibrium of Case 1 dominates any equilibrium of Case 2 (and vice versa) then we get a weaker but similar result. Case 1 dominates Case 2 when only revelation is a refined equilibrium of Case 1, i.e., when $\theta^V > \theta_N^*$. Case 2 dominates Case 1 when only polarization is a refined equilibrium of Case 1, i.e., when $\theta^V < \theta_R^*$. When both revelation and polarization are refined equilibria of Case 1, i.e., when $\theta_R^* \leq \theta^V \leq \theta_N^*$, a comparison is not possible.

Then we show that, in each of the two cases, voters are better off if they have more accurate information, i.e., if θ^V is higher. More precisely we compare the optimal equilibrium for two different values of θ^V (while holding all other parameters fixed) with respect to the ex ante expected utility of voters and show that all voters are better off when θ^V is higher (note that we use the same restrictions on equilibria as above). To see this, first note that for a fixed strategy profile the ex ante expected total utility of each voter is increasing in θ^V . This follows from the proof of Proposition 5.1. With this observation the conclusion easily follows from the table above. In Case 2 pandering is always the optimal equilibrium and thus the conclusion is trivial. In Case 1 the optimal equilibrium is polarization for low values of θ^V (below θ_R^*) and revelation for high values of θ^V (above θ_R^*). And since revelation dominates polarization we are done. Note that if we use policy utility instead of total utility we get the same result. Also note that, in both cases and with respect to both policy and total utility, we can strengthen the result above so that we do not only consider the optimal equilibrium for each set of parameter values. Let $\theta_1^V > \theta_2^V$. In Case 1 it is easy to see that each equilibrium of the θ_1^V -situation Pareto dominates all equilibria of the θ_2^V -situation. This is not true in Case 2, but we can still strengthen our first result. Let $S_{\theta_i^V}$, $i = 1, 2$, be the set of all equilibrium strategy profiles of the θ_i^V -situation. Then it is easily seen that all profiles in $S_{\theta_2^V} \setminus S_{\theta_1^V}$ are Pareto dominated by each profile in $S_{\theta_1^V}$. So all strategy profiles that are part of an equilibrium in the θ_2^V -situation but not in the θ_1^V -situation are Pareto dominated by each equilibrium strategy in the θ_1^V -situation.

Finally we consider the welfare effect of a change in γ . It is obvious that each of the three strategy profiles considered above provide a higher ex ante expected

total utility for each voter when γ is increased. So if an increase in γ does not lead to a different equilibrium outcome then it makes all voters better off. However, in Case 1 an increase in γ may lead to a decrease in welfare for all voters in the sense that the optimal equilibrium after the increase gives all voters a lower ex ante expected utility. To see this, consider a situation where θ^V is just above θ_R^* such that we have revelation. Suppose γ is increased slightly such that the level of voter information is below the new cut-off value for existence of a revealing equilibrium. Then the equilibrium outcome changes to polarization which means that there is a downward jump in welfare for each voter. Thus we see that making each candidate better qualified (in the state where he has a quality advantage) can make all voters worse off.

6 Discussion

We have analyzed a model of electoral competition under uncertainty where candidates are better informed than voters. Candidates were assumed to be office-motivated and to differ only in state-dependent quality. The office-motivation creates an electoral pressure for the candidates to converge on the median of the true state and thus reveal this state to the voters. However, because of the difference in quality there is always one candidate who also has an incentive to choose a policy position such that voters cannot infer the true state. This is the central strategic aspect of the model.

If voters' information is unknown to the candidates when they take positions (Case 1) then, in refined equilibrium, candidates will reveal the state by converging on the true median if the voters' information is sufficiently accurate. If the information of the voters is not sufficiently accurate then each candidate will announce the median position of the state in which he has a quality advantage. Thus the candidates will be polarized and the voters are not able to infer the state.

We also considered the situation where candidates know voters' information when they take positions (Case 2). In this case a refined equilibrium where the candidates' positions reveal the state does not exist. Thus voters always have to rely on their own information when deciding who to vote for. Candidates will either pander by converging on the median of the state that is most likely given the voters' signal or be polarized (or something in between).

With respect to welfare we saw that, from an ex ante perspective, it is optimal for all voters that candidates reveal the state by converging on the true median. Furthermore, pandering is better than polarization. Therefore, when voters are sufficiently well informed it is better for them that candidates do not know their information when they take positions (Case 1). But when voters are poorly informed then it is the other way around, they are all better off if candidates know

which signal they have received (Case 2).

A number of interesting insights about the functioning of representative democracy when candidates are better informed than voters follow from our analysis. First, our results show that policy motivated candidates are not necessary for electoral competition to be inefficient with respect to making all information available to the electorate. As we have seen, differences in state-dependent quality can make fully office-motivated candidates play non-revealing strategies. However, if voters are reasonably well informed and candidates are uncertain about voters' information when they take positions then electoral competition is efficient in this respect. So our results point to the importance of an informed electorate and conditions that make candidates take positions at a point in time where they are uncertain about what information voters will have on election day.

Our analysis also revealed that, loosely speaking, polarization is more likely when the electorate is poorly informed. In Case 1 polarization is the only possible outcome for low values of θ^V , revelation is the only outcome for high values of θ^V , and for medium values both outcomes are possible. In Case 2 (full) polarization is a possibility for low values of θ^V . As we increase θ^V less and less polarization is possible and eventually pandering (or something very close) is the only possibility. So a general prediction of our model is that there is an inverse relationship between the level of polarization and the level of voter information.

It is also interesting to note that, in Case 1, if γ increases then it takes more accurate voter information to get existence of a revealing equilibrium. So an election with two candidates who have substantially different skills (high γ) is more likely to lead to a bad policy choice than an election with two reasonably similar candidates (low γ). And it can even happen that an increase in γ makes all voters worse off with respect to ex ante total utility.

Finally note that the optimal equilibrium of the case where candidates know voters' information when they take positions is always pandering. Therefore it is somewhat counterintuitive that this case is optimal when the electorate is not too well informed. But this is true, of course, because the optimal equilibrium outcome of Case 1 is better than pandering when voters are well informed and worse when they are poorly informed. So in this respect we have the counterintuitive result that pandering is optimal if and only if the electorate not well informed.

7 References

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8 Appendix A

Proof of Proposition 4.3.

First we show that $\theta^V \geq \theta_R^* \Rightarrow$ existence.

By Proposition 4.2 we know that the candidate strategies must be given by $\hat{x}^L(\omega) = \hat{x}^H(\omega) = x_m^*(\omega)$, $\omega = L, H$. Consider a belief function $\hat{\mu}_L$ that satisfies

$$\begin{aligned}
\hat{\mu}_L(x_m^*(L), x, l) &= \hat{\mu}_L(x_m^*(L), x, h) = 1 \text{ for all } x \neq x_m^*(H); \\
\hat{\mu}_L(x_m^*(H), x, l) &= \hat{\mu}_L(x_m^*(H), x, h) = 0 \text{ for all } x \neq x_m^*(L); \\
\hat{\mu}_L(x, x_m^*(L), l) &= \hat{\mu}_L(x, x_m^*(L), h) = 1 \text{ for all } x \neq x_m^*(H); \\
\hat{\mu}_L(x, x_m^*(H), l) &= \hat{\mu}_L(x, x_m^*(H), h) = 0 \text{ for all } x \neq x_m^*(L); \\
\hat{\mu}_L(x_m^*(L), x_m^*(H), l) &= 1, \mu_L(x_m^*(L), x_m^*(H), h) = 0; \\
\hat{\mu}_L(x_m^*(H), x_m^*(L), l) &= \mu_L(x_m^*(H), x_m^*(L), h) = \frac{1}{2}.
\end{aligned}$$

We claim that $\hat{x}^L(\omega)$, $\hat{x}^H(\omega)$, $\hat{\mu}_L(x^L, x^H, \omega^V)$ constitutes an ε -unprejudiced revealing equilibrium when $\theta^V \geq \theta_R^*$. Obviously Bayes rule is satisfied on the (prospective) equilibrium path. Furthermore, if we have an equilibrium then it is obviously ε -unprejudiced (in fact it is unprejudiced). Thus we just have to check the optimality of each candidate's strategy. First consider the strategies in state L . Obviously neither candidate can gain by deviating to some $x \neq x_m^*(H)$ so we just have to check that neither candidate can profitably deviate to $x_m^*(H)$. In equilibrium Candidate L wins with probability

$$\Pr(\omega^V = l | \omega = L) \Pr[\delta > -\gamma] + \Pr(\omega^V = h | \omega = L) \Pr[\delta > \gamma] = \frac{1}{2} + \gamma\sigma.$$

Thus Candidate H wins with probability $\frac{1}{2} - \gamma\sigma$. If Candidate L deviates to $x_m^*(H)$ then his probability of winning is $\frac{1}{2}$ so this deviation is never profitable for him. If Candidate H deviates to $x_m^*(H)$ then his probability of winning is

$$\begin{aligned}
\Pr(\omega^V = l | \omega = L) \Pr[\delta < -(D + \gamma)] + \Pr(\omega^V = h | \omega = L) \Pr[\delta < \gamma + D] \\
= \frac{1}{2} + (1 - 2\theta^V)(\gamma + D)\sigma.
\end{aligned}$$

So the deviation is not profitable if (and only if)

$$\frac{1}{2} + (1 - 2\theta^V)(\gamma + D)\sigma \leq \frac{1}{2} - \gamma\sigma.$$

This inequality is equivalent to

$$\theta^V \geq \theta_R^*.$$

By symmetry it follows that if neither candidate has a profitable deviation in state L then this is also the case in state H . Thus the equilibrium conditions are satisfied if $\theta^V \geq \theta_R^*$.

Finally we show that existence $\Rightarrow \theta^V \geq \theta_R^*$.

Suppose there exists an ε -unprejudiced revealing equilibrium. We know from Proposition 4.2 that each candidate's strategy must be to announce the median of the true state. Two necessary conditions for equilibrium are that Candidate H cannot profitably deviate to $x_m^*(H)$ in state L and that Candidate L cannot profitably deviate to $x_m^*(H)$ in state H . Let $\hat{\mu}_L$ be the equilibrium belief function and define

$$\hat{\mu}_L^l = \hat{\mu}_L(x_m^*(L), x_m^*(H), l) \text{ and } \hat{\mu}_L^h = \hat{\mu}_L(x_m^*(L), x_m^*(H), h).$$

Then the necessary conditions can be written

$$\begin{aligned} & \Pr(\omega^V = l | \omega = L) \left(\frac{1}{2\sigma} - \hat{\mu}_L^l(\gamma + D) + (1 - \hat{\mu}_L^l)(\gamma + D) \right) \sigma \\ & + \Pr(\omega^V = h | \omega = L) \left(\frac{1}{2\sigma} - \hat{\mu}_L^h(\gamma + D) + (1 - \hat{\mu}_L^h)(\gamma + D) \right) \sigma \leq \frac{1}{2} - \gamma\sigma \end{aligned}$$

and

$$\begin{aligned} & \Pr(\omega^V = h | \omega = H) \left(\frac{1}{2\sigma} - (1 - \hat{\mu}_L^h)(\gamma + D) + \hat{\mu}_L^h(\gamma + D) \right) \sigma \\ & + \Pr(\omega^V = l | \omega = H) \left(\frac{1}{2\sigma} - (1 - \hat{\mu}_L^l)(\gamma + D) + \hat{\mu}_L^l(\gamma + D) \right) \sigma \leq \frac{1}{2} - \gamma\sigma. \end{aligned}$$

Thus it suffices to show that if both the two inequalities above are satisfied then we have $\theta^V \geq \theta_R^*$. By adding the two inequalities and a bit of algebra we get

$$(\hat{\mu}_L^l - \hat{\mu}_L^h)(2\theta^V - 1)(\gamma + D) \geq \gamma.$$

Thus we see that $\hat{\mu}_L^l > \hat{\mu}_L^h$ and then it follows that

$$(2\theta^V - 1)(\gamma + D) \geq \frac{\gamma}{(\hat{\mu}_L^l - \hat{\mu}_L^h)} \geq \gamma.$$

Rearranging this inequality we get $\theta^V \geq \theta_R^*$. \square

Proof of observation on page 14.

Let $\hat{x}^L, \hat{x}^H, \hat{\mu}_L(x^L, x^H, \omega^V)$ be a non-revealing equilibrium with $(\hat{x}^L, \hat{x}^H) \notin [x_m^*(L), x_m^*(H)]^2$. Define the strategy profile $(\tilde{x}^L, \tilde{x}^H) \in [x_m^*(L), x_m^*(H)]^2$ by

$$\tilde{x}^i = \left\{ \begin{array}{ll} x_m^*(L) & \text{if } \hat{x}^i < x_m^*(L) \\ \hat{x}^i & \text{if } x_m^*(L) \leq \hat{x}^i \leq x_m^*(H) \\ x_m^*(H) & \text{if } x_m^*(H) < \hat{x}^i \end{array} \right\} \text{ for } i = L, H.$$

Define $\tilde{\mu}_L$ by, for each value of ω^V ,

$$\begin{aligned} \tilde{\mu}_L(\tilde{x}^L, \tilde{x}^H, \omega^V) &= \hat{\mu}_L(\hat{x}^L, \hat{x}^H, \omega^V); \\ \tilde{\mu}_L(\tilde{x}^L, x, \omega^V) &= \hat{\mu}_L(\hat{x}^L, x, \omega^V) \text{ for all } x \neq \tilde{x}^H; \\ \tilde{\mu}_L(x, \tilde{x}^H, \omega^V) &= \hat{\mu}_L(x, \hat{x}^H, \omega^V) \text{ for all } x \neq \tilde{x}^L. \end{aligned}$$

We claim that $\tilde{x}^L, \tilde{x}^H, \tilde{\mu}_L(x^L, x^H, \omega^V)$ is a non-revealing equilibrium. Obviously, Bayes rule is satisfied on the equilibrium path. To check that neither candidate has a profitable deviation first define

$$D_i = \text{dist}(\hat{x}^i, [x_m^*(L), x_m^*(H)]), \quad i = L, H.$$

Let \hat{P}_i^{Eq} denote Candidate i 's equilibrium probability of winning in the original equilibrium. It is straightforward to check that Candidate i 's probability of winning in the new situation is

$$\hat{P}_i^{Eq} + (D_i - D_j)\sigma, \quad j \neq i.$$

Let $\hat{P}_i(x)$ denote Candidate i 's probability of winning when deviating to $x \neq \hat{x}^i$ in the original equilibrium. It is straightforward to check that Candidate i 's probability of winning by deviating to x in the new situation is (if positive)

$$\hat{P}_i(x) - D_j\sigma, \quad j \neq i.$$

So a deviation by i to $x \neq \hat{x}^i$ is not profitable if

$$\hat{P}_i(x) \leq \hat{P}_i^{Eq} + D_i\sigma.$$

This is obviously true since $(\hat{x}^L, \hat{x}^H), \hat{\mu}_L$ is an equilibrium. If $\tilde{x}^i \neq \hat{x}^i$ we also have to consider deviations to \hat{x}^i . After such a deviation the belief of the voters' is exactly as when they observe $(\tilde{x}^L, \tilde{x}^H)$, i.e., it is given by Bayesian updating based on the value of ω^V . And since the deviating candidate is moving from $x_m^*(L)$ to $\hat{x}^i < x_m^*(L)$ or from $x_m^*(H)$ to $\hat{x}^i > x_m^*(H)$ it follows that such a deviation cannot be profitable. \square

Proof of Proposition 4.4.

1. Consider strategies \hat{x}^L , \hat{x}^H and a belief function $\hat{\mu}_L(x^L, x^H, \omega^V)$ satisfying

$$\begin{aligned}\hat{\mu}_L(\hat{x}^L, \hat{x}^H, l) &= \theta^V \text{ and } \hat{\mu}_L(\hat{x}^L, \hat{x}^H, h) = 1 - \theta^V; \\ \hat{\mu}_L(\hat{x}^L, x, \omega^V) &= 1 \text{ for all } x \neq \hat{x}^H, \omega^V = l, h; \\ \hat{\mu}_L(x, \hat{x}^H, \omega^V) &= 0 \text{ for all } x \neq \hat{x}^L, \omega^V = l, h.\end{aligned}$$

This belief function satisfies Bayes rule on the equilibrium path and no other belief function makes it less profitable for either candidate to deviate. Thus it suffices to show that \hat{x}^L , \hat{x}^H , $\hat{\mu}_L(x^L, x^H, \omega^V)$ is an equilibrium if and only if the condition on θ^V from the proposition is satisfied. It is easy to see that no candidate can profitably deviate if and only if Candidate H cannot profitably deviate to $x_m^*(L)$ in state L and Candidate L cannot profitably deviate to $x_m^*(H)$ in state H . (If $\hat{x}^H = x_m^*(L)$ ($\hat{x}^L = x_m^*(H)$) we just have to check that Candidate H (L) cannot profitably deviate to a position very close to $x_m^*(L)$ ($x_m^*(H)$)).

In state L Candidate H 's equilibrium probability of winning is

$$\frac{1}{2} - (2\theta^V - 1)^2(\gamma + (\hat{x}^H - \hat{x}^L))\sigma.$$

By deviating to $x_m^*(L)$ Candidate H wins with probability

$$\frac{1}{2} - (\gamma - (\hat{x}^L - x_m^*(L)))\sigma.$$

(If $\hat{x}^H = x_m^*(L)$ then he can win with a probability that is arbitrarily close to this number by deviating to a position x that is sufficiently close to $x_m^*(L)$). Thus the deviation is not profitable if and only if

$$\frac{1}{2} - (2\theta^V - 1)^2(\gamma + (\hat{x}^H - \hat{x}^L))\sigma \geq \frac{1}{2} - (\gamma - (\hat{x}^L - x_m^*(L)))\sigma,$$

which is equivalent to

$$\theta^V \leq \frac{1}{2} \left(1 + \sqrt{\frac{\gamma - (\hat{x}^L - x_m^*(L))}{(\gamma + (\hat{x}^H - \hat{x}^L))}} \right).$$

Analogously we get that Candidate L does not have a profitable deviation if

$$\theta^V \leq \frac{1}{2} \left(1 + \sqrt{\frac{\gamma - (x_m^*(H) - \hat{x}^H)}{(\gamma + (\hat{x}^H - \hat{x}^L))}} \right).$$

Thus we have an equilibrium if and only if θ^V is below the minimum of the two cut-off values above. The minimum is precisely the cut-off value from the statement.

2. First note that if (\hat{x}^L, \hat{x}^H) is an equilibrium strategy profile and $\hat{x}^L - \hat{x}^H \neq \gamma$ then each candidate will, in one state, win with a probability strictly lower than $\frac{1}{2}$. Also note that at least one of the candidates has a deviation that will give him a win with probability $\frac{1}{2}$ no matter what the state is and no matter what voters believe if they observe this deviation: If $\hat{x}^L - x_m^*(L) > \gamma$ then candidate H can deviate to $\hat{x}^L - \gamma$, if $x_m^*(H) - \hat{x}^H > \gamma$ then candidate L can deviate to $\hat{x}^H + \gamma$. Thus (\hat{x}^L, \hat{x}^H) with $\hat{x}^L - \hat{x}^H \neq \gamma$ cannot be an equilibrium strategy profile.

Then consider (\hat{x}^L, \hat{x}^H) with $\hat{x}^L - \hat{x}^H = \gamma$ and a voter belief function $\hat{\mu}_L$ satisfying

$$\begin{aligned}\hat{\mu}_L(\hat{x}^L, \hat{x}^H, l) &= \theta^V \text{ and } \hat{\mu}_L(\hat{x}^L, \hat{x}^H, h) = 1 - \theta^V; \\ \hat{\mu}_L(\hat{x}^L, x, \omega^V) &= 0 \text{ for all } x < \hat{x}^H, \omega^V = l, h; \\ \hat{\mu}_L(\hat{x}^L, x, \omega^V) &= 1 \text{ for all } x > \hat{x}^H, \omega^V = l, h; \\ \hat{\mu}_L(x, \hat{x}^H, \omega^V) &= 0 \text{ for all } x < \hat{x}^L, \omega^V = l, h; \\ \hat{\mu}_L(x, \hat{x}^H, \omega^V) &= 1 \text{ for all } x > \hat{x}^L, \omega^V = l, h.\end{aligned}$$

In this situation each candidate wins with probability $\frac{1}{2}$ in each state. If a candidate deviates then he wins with a probability that is strictly smaller than $\frac{1}{2}$. Therefore we have an equilibrium. \square

Proof of Proposition 4.7.

1. The necessity of the first condition follows from monotonicity (see the remarks above the proposition). And then the necessity of the second condition follows from part 1. of the result in Appendix B. To show that the conditions are sufficient we have to show that $((x_m^*(L), \hat{x}^L(h)), (\hat{x}^H(l), x_m^*(H)))$ can be supported as an equilibrium by a *monotone* belief function. Let $\hat{\mu}_L$ be a monotone belief function satisfying the following conditions (none of these conditions violate monotonicity).

$$\begin{aligned}\hat{\mu}_L(x_m^*(L), \hat{x}^H(l), l) &= \theta^V \text{ and } \hat{\mu}_L(\hat{x}^L(h), x_m^*(H), h) = 1 - \theta^V; \\ \hat{\mu}_L(x_m^*(L), x, l) &= 1 \text{ for all } x < \hat{x}^H(l); \\ \hat{\mu}_L(x_m^*(L), x, l) &= \theta^V \text{ for all } x > \hat{x}^H(l); \\ \hat{\mu}_L(\hat{x}^L(h), x, h) &= 1 - \theta^V \text{ for all } x \neq x_m^*(H); \\ \hat{\mu}_L(x, x_m^*(H), h) &= 1 - \theta^V \text{ for all } x < \hat{x}^L; \\ \hat{\mu}_L(x, x_m^*(H), h) &= 0 \text{ for all } x > \hat{x}^L; \\ \hat{\mu}_L(x, \hat{x}^H(l), l) &= \theta^V \text{ for all } x \neq x_m^*(L).\end{aligned}$$

It is easy to see that if Candidate H cannot profitably deviate to $x_m^*(L)$ when $\omega^V = l$ (if $\hat{x}^H(l) \neq x_m^*(L)$) and Candidate L cannot profitably deviate to $x_m^*(H)$ when $\omega^V = h$ (if $\hat{x}^H(h) \neq x_m^*(H)$) then neither candidate has any profitable

deviations. Straightforward calculations show that neither of these two deviations are profitable if θ^V is below the upper bound stated in the proposition.

2. Suppose $((\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h))) \notin [x_m^*(L), x_m^*(H)]^4$ is the strategy profile of a monotone non-revealing equilibrium. Consider the profile where, for each i and ω^V , $\hat{x}^i(\omega^V)$ is replaced by $x_m^*(L)$ if $\hat{x}^i(\omega^V) < x_m^*(L)$ and by $x_m^*(H)$ if $x_m^*(H) < \hat{x}^i(\omega^V)$. Denote this profile $((\tilde{x}^L(l), \tilde{x}^L(h)), (\tilde{x}^H(l), \tilde{x}^H(h)))$. Because of monotonicity we must have $\tilde{x}^L(l) = x_m^*(L)$ and $\tilde{x}^H(h) = x_m^*(H)$. Let $\tilde{\mu}_L$ be a monotone belief function satisfying the same conditions as $\hat{\mu}_L$ in the proof of the first statement (with $\hat{x}^L(h)$ and $\hat{x}^H(l)$ replaced by $\tilde{x}^L(h)$ and $\tilde{x}^H(l)$). We then claim that if Candidate i can profitably deviate from $\hat{x}^i(\omega^V)$ when the voters' belief is given by $\tilde{\mu}_L$ then the same deviation would also be profitable in the original equilibrium, which is a contradiction. Candidate H has a profitable deviation from $\tilde{x}^H(\omega^V)$ when the voters' belief is given by $\tilde{\mu}_L$ if and only if it is profitable for him to deviate to $x_m^*(L)$ when $\omega^V = l$. Suppose this is the case. Using notation similar to that in the proof of the observation on page 14, Candidate H 's probability of winning from announcing $\tilde{x}^H(l)$ when $\omega^V = l$ is

$$\hat{P}_H^{Eq}(l) + (D_H(l) - D_L(l))\sigma.$$

Candidate H 's probability of winning after deviating to $x_m^*(L)$ is at most

$$\hat{P}_H(x_m^*(L), l) - D_L(l)\sigma.$$

Thus we see that if the deviation is profitable then we must have

$$\hat{P}_H(x_m^*(L), l) > \hat{P}_H^{Eq}(l),$$

which means that in the original equilibrium a deviation by Candidate H to $x_m^*(L)$ is profitable, which is a contradiction. Analogously we also get a contradiction if we assume that Candidate L can profitably deviate from $\tilde{x}^L(h)$ to $x_m^*(H)$ when $\omega^V = h$. Thus $(\tilde{x}^L(\omega^V), \tilde{x}^H(\omega^V), \tilde{\mu}_L(x^L, x^H, \omega^V))$ is a monotone non-revealing equilibrium. \square

Proof of Proposition 5.1.

1. First we consider the median voter. The median voter's ex ante expected policy utility from the revelation strategy profile is obviously equal to zero because both candidates are converging on the true median in both states. The median voter's ex ante expected policy utility from the pandering profile is

$$(\Pr(L, l) + \Pr(H, h))(0) + (\Pr(L, h) + \Pr(H, l))(-D) = -(1 - \theta^V)D.$$

Thus we immediately see that the revelation profile makes the median voter better off than the pandering profile.

Let δ_{pol}^* denote the δ -value for which the median voter switches his vote when the candidates are polarized and $\omega^V = l$ (the δ -value for which the median voter switches his vote when $\omega^V = h$ is then $-\delta_{pol}^*$). The median voter's ex ante expected policy utility from the polarization profile can then be written

$$\begin{aligned} & \Pr(L, l) \Pr(\delta < \delta_{pol}^*)(-D) + \Pr(H, \omega h) \Pr(\delta > -\delta_{pol}^*)(-D) \\ & + \Pr(L, h) \Pr(\delta < -\delta_{pol}^*)(-D) + \Pr(H, l) \Pr(\delta > \delta_{pol}^*)(-D) \\ = & \theta^V \Pr(\delta < \delta_{pol}^*)(-D) + (1 - \theta^V) \Pr(\delta > \delta_{pol}^*)(-D). \end{aligned}$$

Since $\delta_{pol}^* = (1 - 2\theta^V)(D + \gamma)$ we have

$$\begin{aligned} \Pr(\delta > \delta_{pol}^*) &= \left(\frac{1}{2\sigma} + (2\theta^V - 1)(D + \gamma)\right)\sigma \\ &= \frac{1}{2} + (2\theta^V - 1)(D + \gamma)\sigma. \end{aligned}$$

Thus the ex ante expected utility becomes

$$D((2\theta^V - 1)^2(D + \gamma)\sigma - \frac{1}{2}).$$

Since $D + \gamma < \frac{1}{2\sigma}$, this number is strictly smaller than

$$D((2\theta^V - 1)^2\frac{1}{2} - \frac{1}{2}) = -2\theta^V(1 - \theta^V)D,$$

which is strictly smaller than $-(1 - \theta^V)D$. Thus the ranking is satisfied for the median voter.

Then consider a voter with $d_i \in (0, D)$. For such a voter the ex ante expected policy utility of the three profiles are

$$\begin{aligned} & -d_i, \\ & -(1 - \theta^V)D - d_i\theta^V, \end{aligned}$$

and

$$D((2\theta^V - 1)^2(D + \gamma)\sigma - \frac{1}{2}) - d_i((2\theta^V - 1)^2(D + \gamma)\sigma + \frac{1}{2}).$$

It is then straightforward to check that the ranking of the proposition is satisfied for all voters with $d_i \in (0, D)$.

For voters with $d_i \geq D$ the ex ante expected policy utility of all three strategy profiles are equal to $-d_i$. So for such voters the profiles are equally good with respect to policy.

2. By the first part of the proposition it suffices to show that the ranking from the proposition holds with respect to ex ante expected *quality utility* ("the γ and δ terms"). With respect to quality all voters have identical preferences, so we do not need to distinguish between different voters.

We first show that revelation gives voters a strictly higher ex ante expected quality utility than pandering. In both of these strategy profiles there is always policy convergence. Thus the voting decision of each voter is based solely on quality utility. When candidates reveal the state then voters make the optimal decision. When candidates are pandering voters do not learn the state and thus, for each combination of ω and ω^V , there are values of δ such that voters do not make the optimal choice (i.e., the optimal choice had they known the state). Therefore revelation must give a higher ex ante expected quality utility.

Finally, we show that pandering dominates polarization with respect to quality utility. Since there is always policy convergence in the pandering profile it follows that, for each combination of values of ω , ω^V , and δ , voters take the optimal decision with respect to quality utility under the constraint that they do not know the state. In the polarization profile there is always policy divergence and therefore, for each combination of values of ω and ω^V , there are values of δ such that the median voter (who decides the outcome of the election) does not make the optimal choice with respect to quality utility under the constraint that he does not know ω . Therefore pandering dominates polarization with respect to ex ante expected quality utility. \square

9 Appendix B

Proposition 9.1 (Non-Rev. Equil. in Case 2: Strategies and Existence)

Let $((\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h))) \in [x_m^*(L), x_m^*(H)]^4$.

1. If $\max\{\hat{x}^L(l) - x_m^*(L), x_m^*(H) - \hat{x}^H(h)\} < \gamma$ then $((\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h)))$ is the strategy profile of a non-revealing equilibrium if and only if

$$\theta^V \leq \frac{1}{2} \left(1 + \min \left\{ \frac{\gamma - (\hat{x}^L(l) - x_m^*(L))}{\gamma + (\hat{x}^H(l) - \hat{x}^L(l))}, \frac{\gamma - (x_m^*(H) - \hat{x}^H(h))}{\gamma + (\hat{x}^H(h) - \hat{x}^L(h))} \right\} \right).$$

2. If $\hat{x}^L(l) - x_m^*(L) \geq \gamma$, $x_m^*(H) - \hat{x}^H(h) < \gamma$ then $((\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h)))$ is the strategy profile of a non-revealing equilibrium if and only if

$$\hat{x}^L(l) - \hat{x}^H(l) = \gamma$$

and

$$\theta^V \leq \frac{1}{2} \left(1 + \frac{\gamma - (x_m^*(H) - \hat{x}^H(h))}{\gamma + (\hat{x}^H(h) - \hat{x}^L(h))} \right).$$

3. If $\hat{x}^L(l) - x_m^*(L) < \gamma$, $x_m^*(H) - \hat{x}^H(h) \geq \gamma$ then $((\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h)))$ is the strategy profile of a non-revealing equilibrium if and only if

$$\hat{x}^L(h) - \hat{x}^H(h) = \gamma$$

and

$$\theta^V \leq \frac{1}{2} \left(1 + \frac{\gamma - (\hat{x}^L(l) - x_m^*(L))}{\gamma + (\hat{x}^H(l) - \hat{x}^L(l))} \right).$$

4. If $\min\{\hat{x}^L(l) - x_m^*(L), x_m^*(H) - \hat{x}^H(h)\} \geq \gamma$ then $((\hat{x}^L(l), \hat{x}^L(h)), (\hat{x}^H(l), \hat{x}^H(h)))$ is the strategy profile of a non-revealing equilibrium if and only if

$$\hat{x}^L(l) - \hat{x}^H(l) = \hat{x}^L(h) - \hat{x}^H(h) = \gamma.$$

Proof.

1. Consider a belief function $\hat{\mu}_L$ satisfying the following conditions:

$$\begin{aligned} \hat{\mu}_L(\hat{x}^L(l), \hat{x}^H(l), l) &= \theta^V \text{ and } \hat{\mu}_L(\hat{x}^L(h), \hat{x}^H(h), h) = 1 - \theta^V; \\ \hat{\mu}_L(\hat{x}^L(h), x, h) &= 1 \text{ for all } x \neq \hat{x}^H(h); \\ \hat{\mu}_L(\hat{x}^L(l), x, l) &= 1 \text{ for all } x \neq \hat{x}^H(l); \\ \hat{\mu}_L(x, \hat{x}^H(h), h) &= 0 \text{ for all } x \neq \hat{x}^L(h); \\ \hat{\mu}_L(x, \hat{x}^H(l), l) &= 0 \text{ for all } x \neq \hat{x}^L(l). \end{aligned}$$

This belief function supports the strategy profile as an equilibrium if and only if Candidate L cannot profitably deviate to $x_m^*(H)$ when $\omega^V = h$ and Candidate H cannot profitably deviate to $x_m^*(L)$ when $\omega^V = l$. These conditions are equivalent to the condition on θ^V stated in the proposition. Finally, if the condition from the proposition is not satisfied then it is easy to see that, no matter how we define out-of-equilibrium beliefs, one of the deviations considered above will be profitable.

2. Use the arguments from 1. when $\omega^V = h$ and the arguments from 4. when $\omega^V = l$.

3. Use the arguments from 1. when $\omega^V = l$ and the arguments from 4. when $\omega^V = h$.

4. Consider a belief function $\hat{\mu}_L$ satisfying the following conditions:

$$\begin{aligned} \hat{\mu}_L(\hat{x}^L(l), \hat{x}^H(l), l) &= \theta^V \text{ and } \hat{\mu}_L(\hat{x}^L(h), \hat{x}^H(h), h) = 1 - \theta^V; \\ \hat{\mu}_L(\hat{x}^L(\omega^V), x, \omega^V) &= 0 \text{ for all } x < \hat{x}^H(\omega^V), \omega^V = l, h; \\ \hat{\mu}_L(\hat{x}^L(\omega^V), x, \omega^V) &= 1 \text{ for all } x > \hat{x}^H(\omega^V), \omega^V = l, h; \\ \hat{\mu}_L(x, \hat{x}^H(\omega^V), \omega^V) &= 0 \text{ for all } x < \hat{x}^L(\omega^V), \omega^V = l, h; \\ \hat{\mu}_L(x, \hat{x}^H(\omega^V), \omega^V) &= 1 \text{ for all } x > \hat{x}^L(\omega^V), \omega^V = l, h. \end{aligned}$$

Such a belief function supports the strategy profile if the conditions stated in the proposition are satisfied.

If $\hat{x}^L(l) - \hat{x}^H(l) < \gamma$ ($\hat{x}^L(h) - \hat{x}^H(h) < \gamma$) then consider a deviation by Candidate H (L) to $\hat{x}^L(l) - \gamma$ ($\hat{x}^H(h) + \gamma$) when $\omega^V = l$ ($\omega^V = h$). After this deviation he wins with probability $\frac{1}{2}$ no matter what the out-of-equilibrium belief of the voters is. Thus it is a profitable deviation.

If $\hat{x}^L(l) - \hat{x}^H(l) > \gamma$ ($\hat{x}^L(h) - \hat{x}^H(h) > \gamma$) then consider a deviation by Candidate L (H) to $\hat{x}^H(l) + \gamma$ ($\hat{x}^L(h) - \gamma$) when $\omega^V = l$ ($\omega^V = h$). After this deviation he wins with probability $\frac{1}{2}$ no matter what the out-of-equilibrium belief of the voters is. Thus it is a profitable deviation. \square